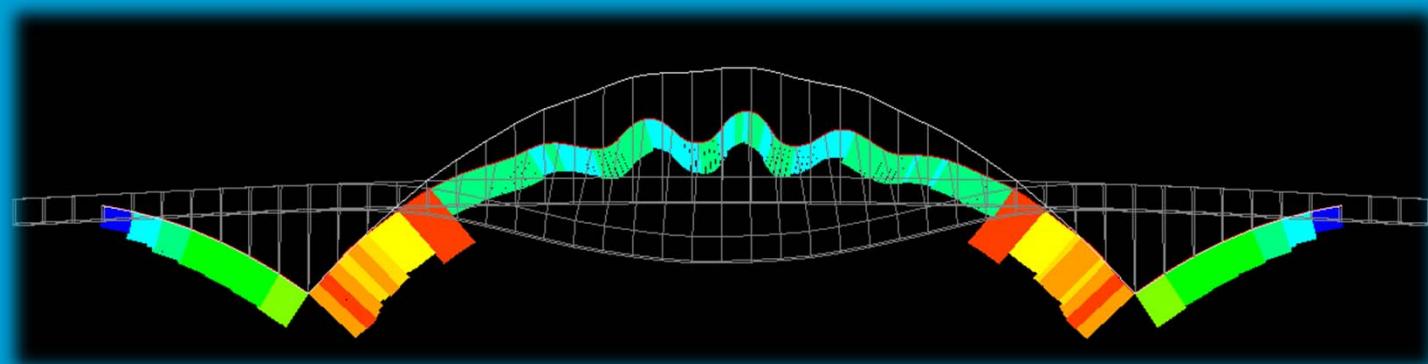


Arch Bridge Buckling Analysis Methods

Benjamin Blasen, P.E.

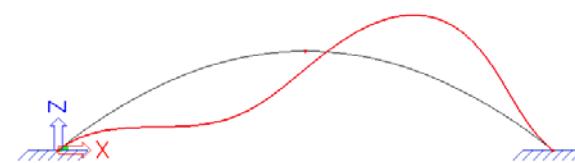
Joe Stith, P.E.



ch2mSM

Outline

- Description of arch buckling analysis methods:
 - AASHTO (simplified Euler Buckling)
 - Eigenvalue Buckling
 - Nonlinear Pushover Analysis
 - Other advanced methods
- Comparison of the buckling capacity.
 - How much does an Eigenvalue Buckling analysis overestimate or underestimate the actual buckling capacity?
- Comparison of the resulting moment magnifiers.
 - If AASHTO method is used for an arch which doesn't meet the criteria, the resulting moment magnifiers may be lower than they should be!



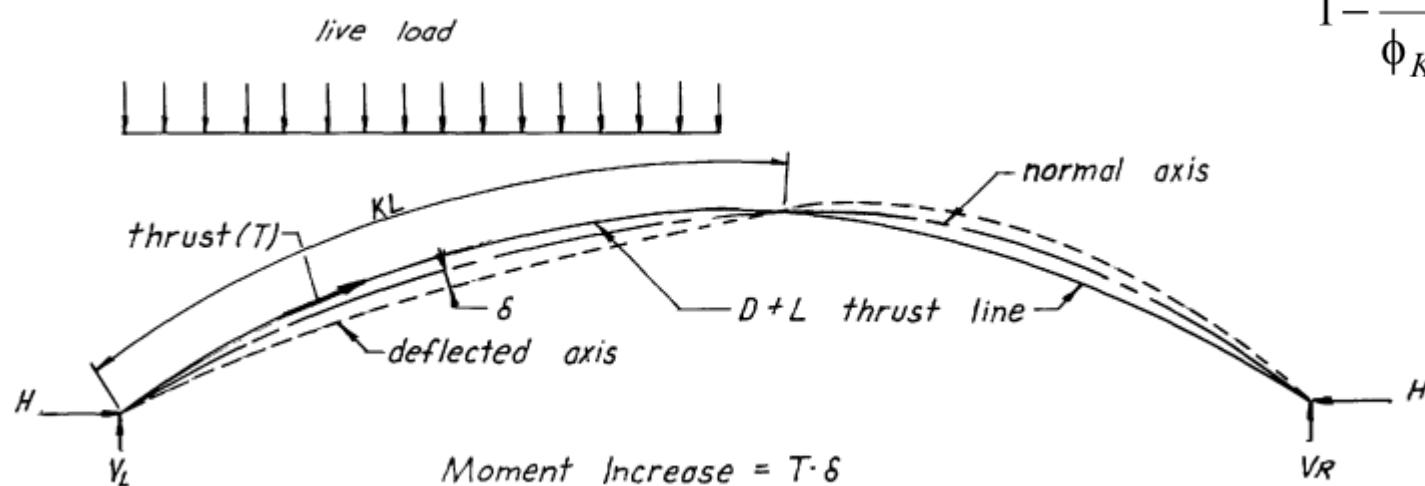
Arch Buckling Methods

Why is Buckling Capacity Needed? ..AASHTO Method

- Arch behavior similar to other axial loaded elements (P-M Interaction)
- Euler Buckling: Stiffness vs. Slenderness
- Static amplification of 1st order moment

$$P_e = \frac{\pi^2 EI}{(K \ell_u)^2}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} \geq 1.0$$



Arch Buckling

- AASHTO 4.5.3.2.2c provides a reference for the unsupported length (l_u) and the effective length factor (K)
- Table does not include 1-Hinge or Tied Arches

$$P_e = \frac{\pi^2 EI}{(K \ell_u)^2}$$

- E, I, and ℓ_u from Arch / Section geometry.

ℓ_u = one-half of the length of the arch rib (ft)
 K = effective length factor specified in Table 4.5.3.2.2c-1

Table 4.5.3.2.2c-1—K Values for Effective Length of Arch Ribs

Rise to Span Ratio	3-Hinged Arch	2-Hinged Arch	Fixed Arch
0.1–0.2	1.16	1.04	0.70
0.2–0.3	1.13	1.10	0.70
0.3–0.4	1.16	1.16	0.72

Arch Buckling

- Limited range of R/S ratio.
- Only three boundary conditions.
- Assumes constant section properties along arch rib!

ℓ_u = one-half of the length of the arch rib (ft)
 K = effective length factor specified in Table 4.5.3.2.2c-1

Table 4.5.3.2.2c-1— K Values for Effective Length of Arch Ribs

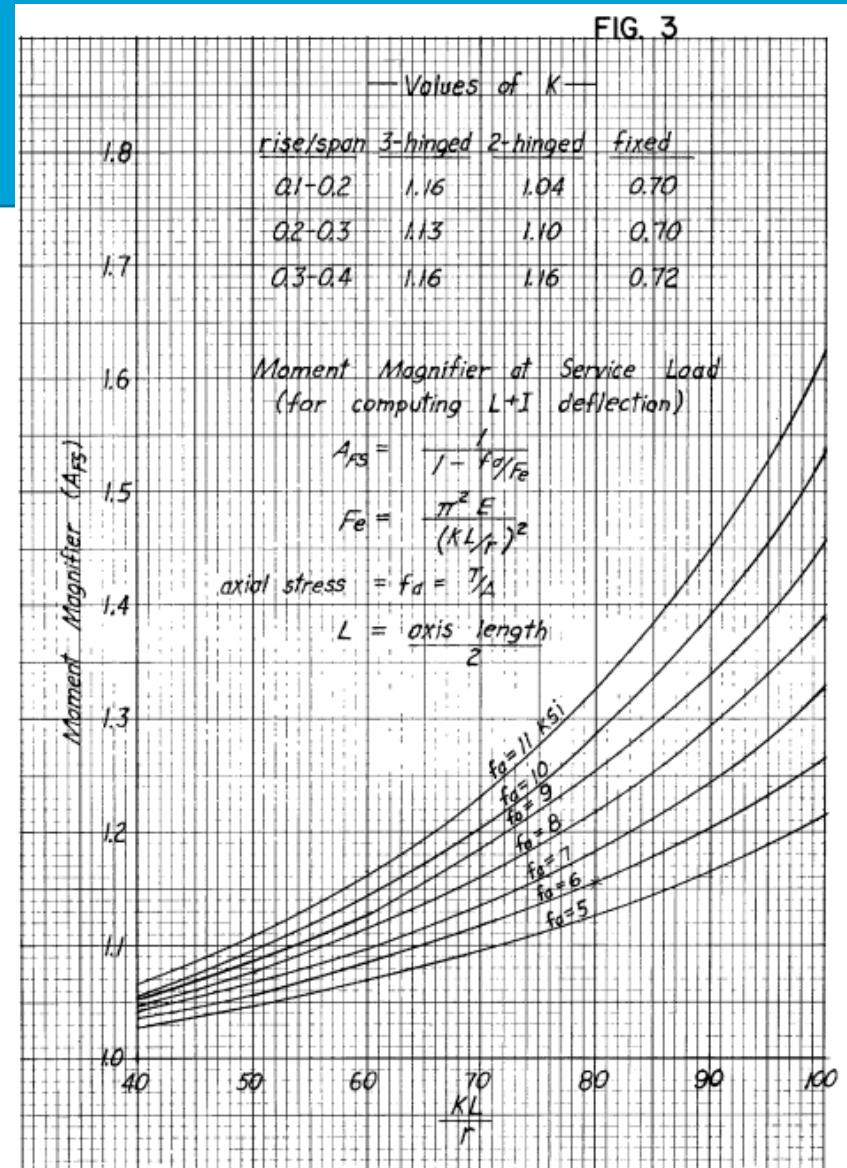
Rise to Span Ratio	3-Hinged Arch	2-Hinged Arch	Fixed Arch
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0.2–0.3	1.13	1.10	0.70
0.3–0.4	1.16	1.16	0.72

Moment Magnification

- AASHTO equations for moment magnification and buckling factors are based on the Nettleton paper *Arch Bridges*

$$A_{Fs} = \frac{1}{1 - \frac{T}{AF_e}} \quad \text{Arch Bridges Eq.}$$

$$\delta_b = \frac{C_m}{1 - \frac{P_u}{\phi_K P_e}} \geq 1.0 \quad \text{AASHTO Eq.}$$



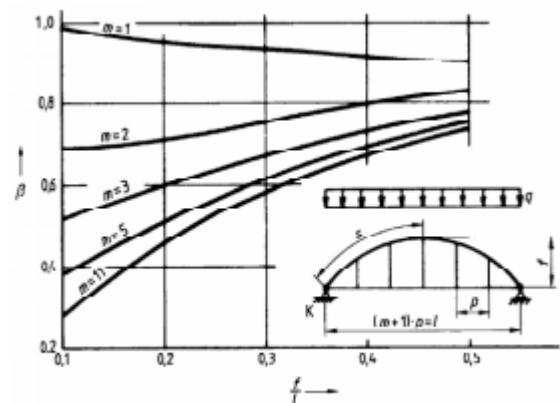
Nettleton, D. A. 1977. *Arch Bridges*. Bridge Division, Office of Engineering, Federal Highway Administration, U.S. Department of Transportation, Washington, DC.

Effective Length Factor

- AASHTO vs Eurocode 3-2 Steel Bridges for 3-Hinged, 2-Hinged, and Fixed
 - Two codes agree on effective length factors ($K = \beta$)

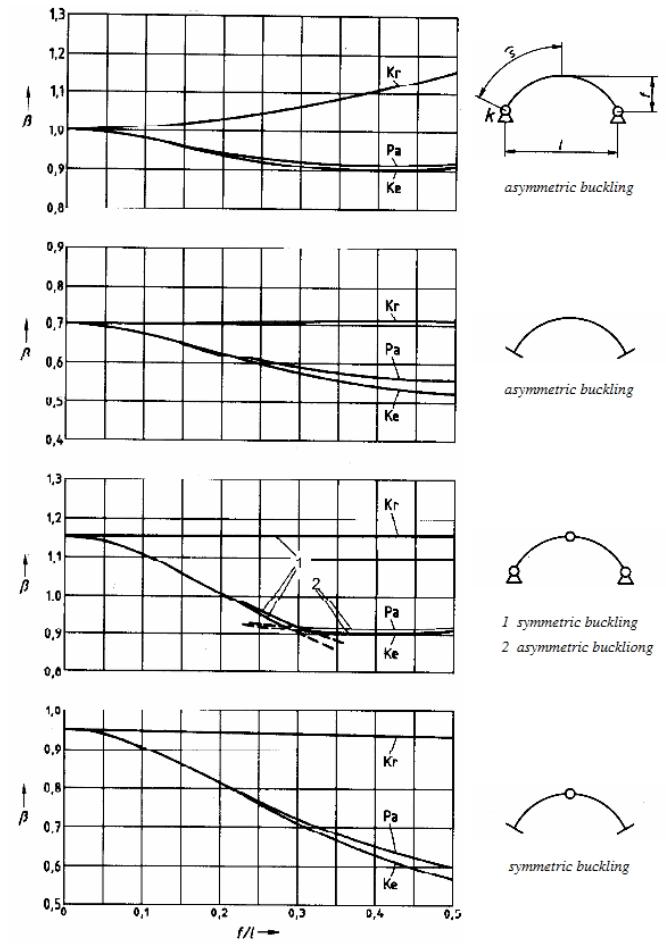
Table 4.5.3.2.2c-1— K Values for Effective Length of Arch Ribs

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0.3–0.4	1.16	1.16	0.72



8

Table D.4: Buckling length factor β for arches for $f/\ell > 0,1$



Pa: parabolic form Ke: chain form
for Pa and Ke the loading is vertical
Kr: circular form
for Kr the radial loading (e.g. hydrostatic pressure) is supposed

(Diagram)

Common Arch Types

Can I use the AASHTO “K” table?



R/S Ratio



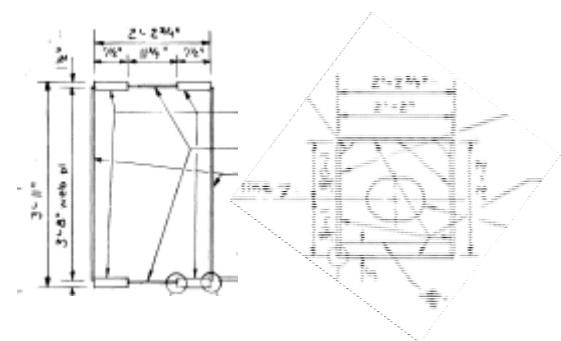
Constant Section



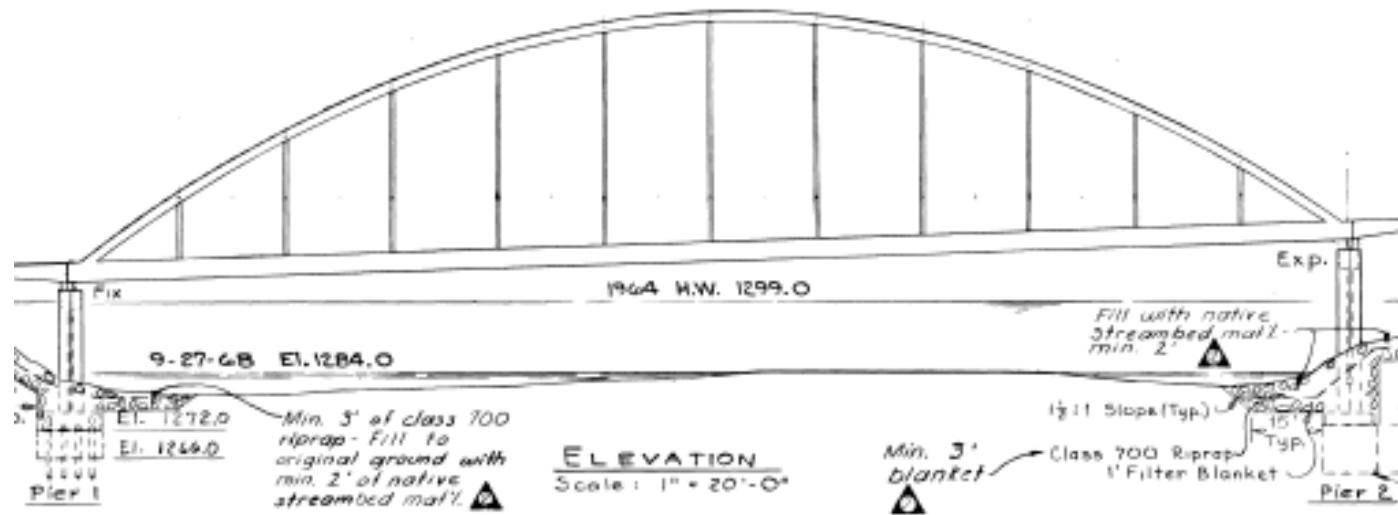
Boundary Conditions

- Moment Tied Arch

- Arch tie carries flexure in addition to tension
 - Floorbeams connect directly to tie
 - Connection between arch and tie



- ❖ John Day River Bridge





R/S Ratio

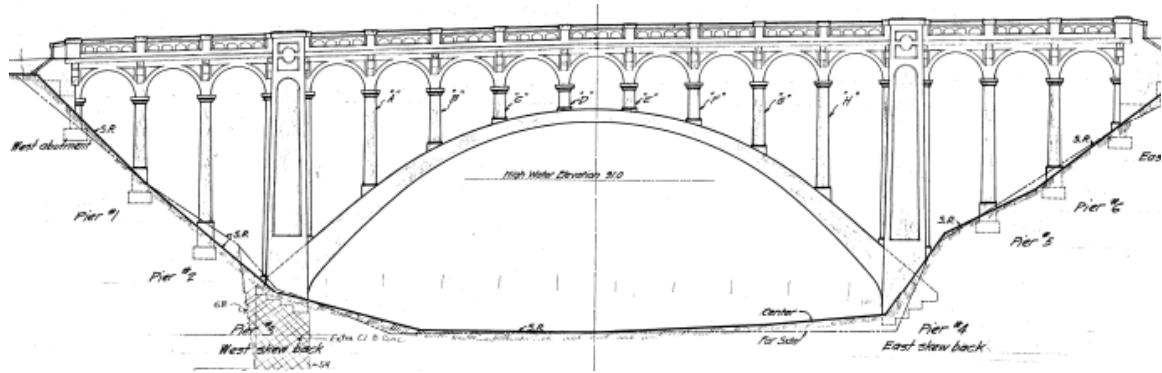


Constant
Section



Boundary
Conditions

- Fixed-Fixed
Concrete Arch



- ❖ Rogue River (Caveman) Bridge
- ❖ Rickreall Creek Bridge
- ❖ Oswego Creek Bridge
- ❖ Rocky Creek Bridge
- ❖ Depoe Bay Bridge
- ❖ Little North Fork Santiam Bridge
- ❖ Crooked River Bridge
- ❖ Cooks Chasm Bridge



R/S Ratio



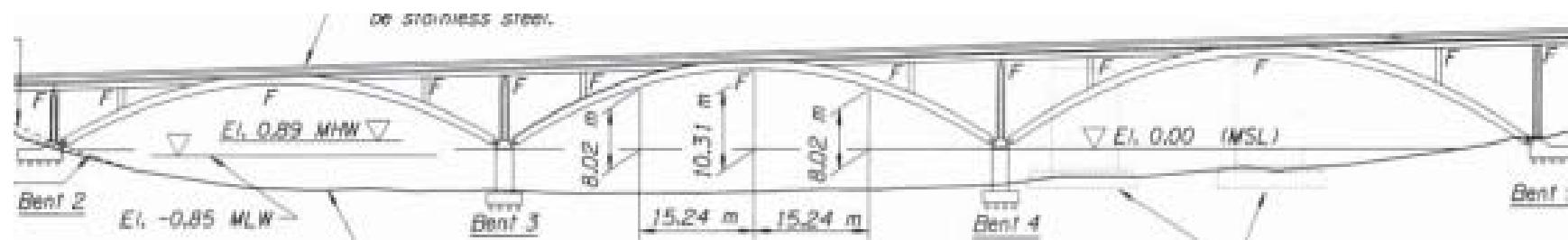
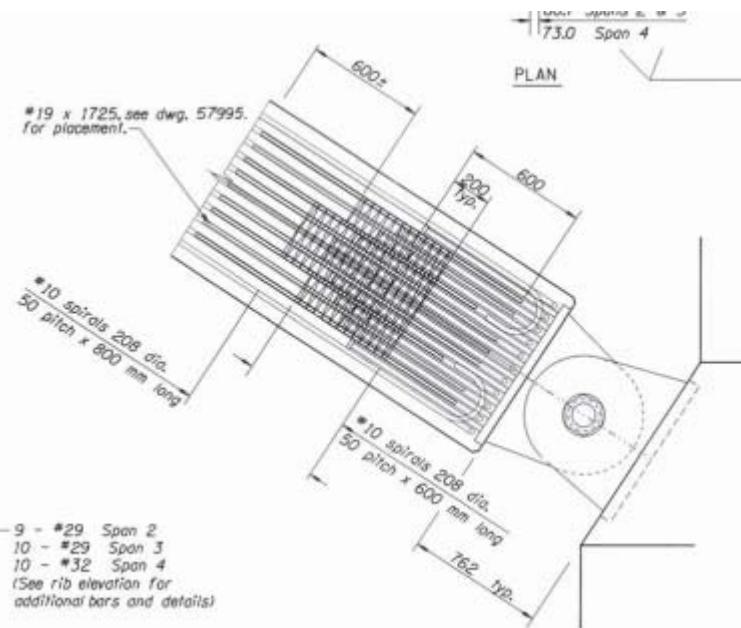
Constant
Section



Boundary
Conditions

- Pinned-Pinned Concrete Arch

❖ Haynes Inlet Slough Bridge





R/S Ratio

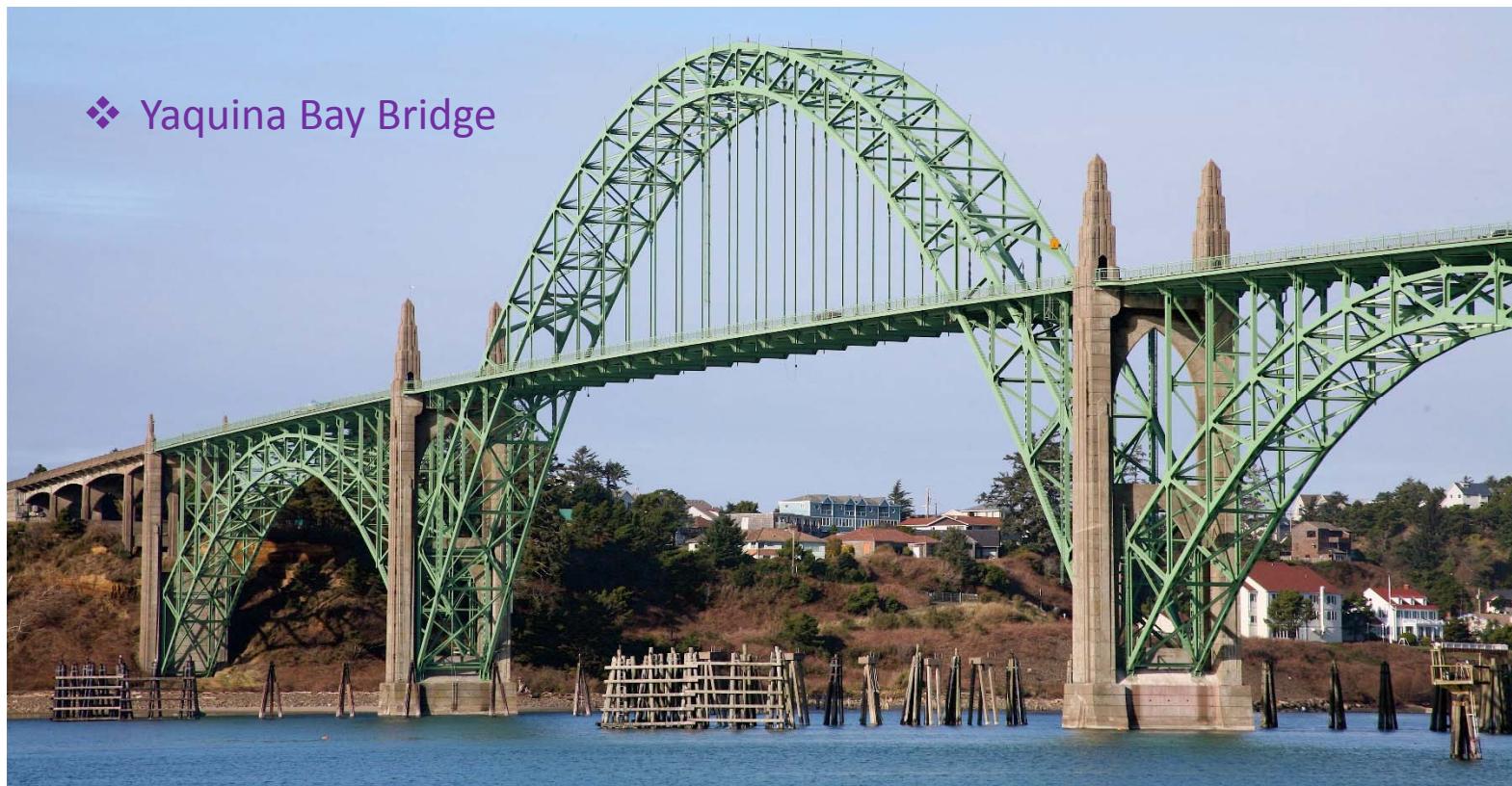


Constant
Section



Boundary
Conditions

- Pinned-Pinned Steel Truss Arch





R/S Ratio



Constant
Section

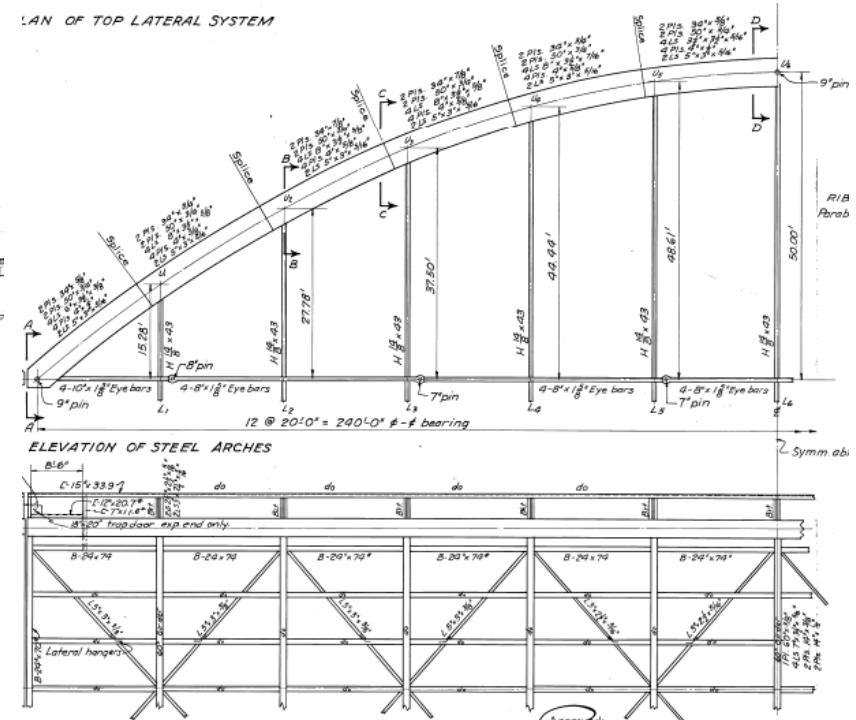
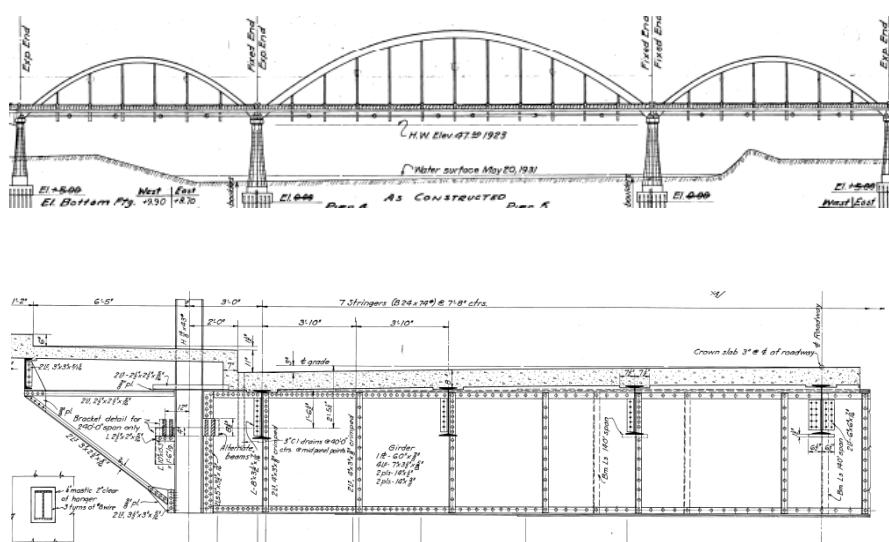


Boundary
Conditions

• Bowstring Arch

- Tension Tie only carries axial load
- Unsymmetrical Flexure carried by Arch
- 3 pin bowstring arch

❖ Clackamas River Bridge





R/S Ratio



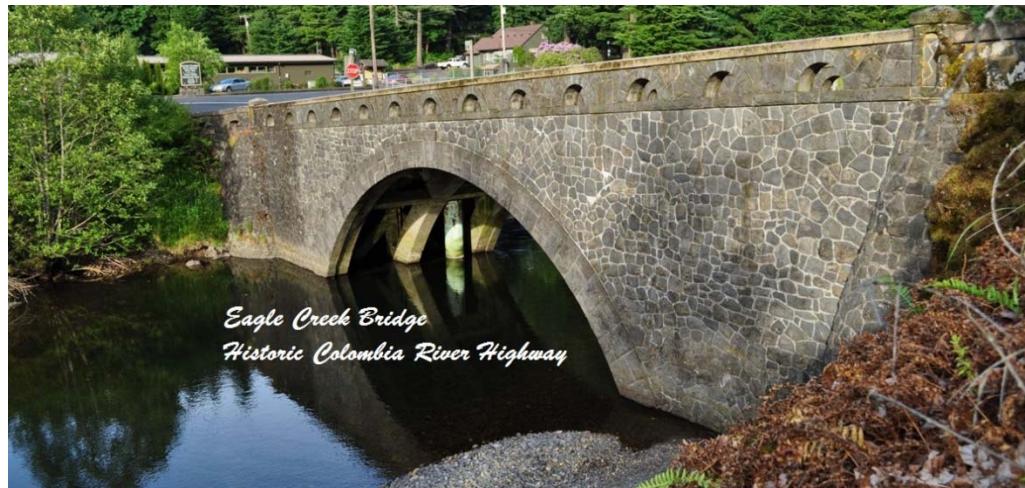
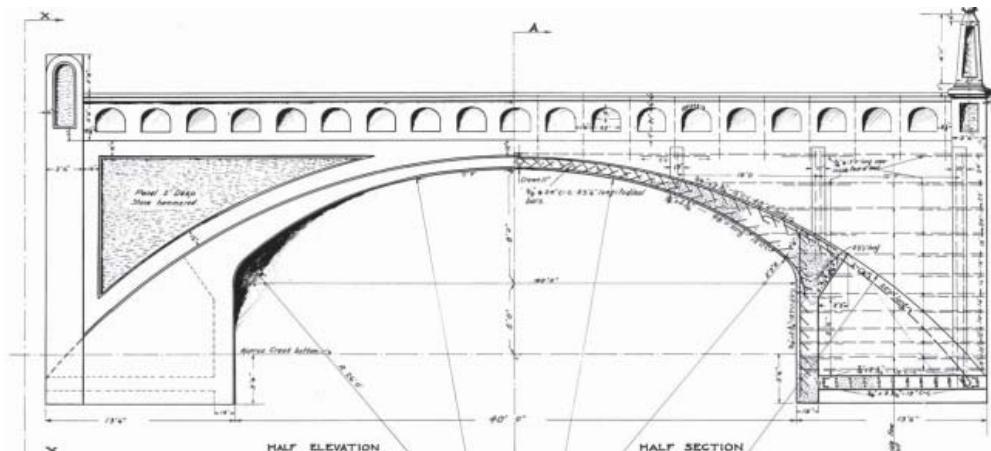
Constant Section



Boundary Conditions

- Filled Spandrel Arch

❖ Multnomah Creek Bridge





R/S Ratio



Constant Section

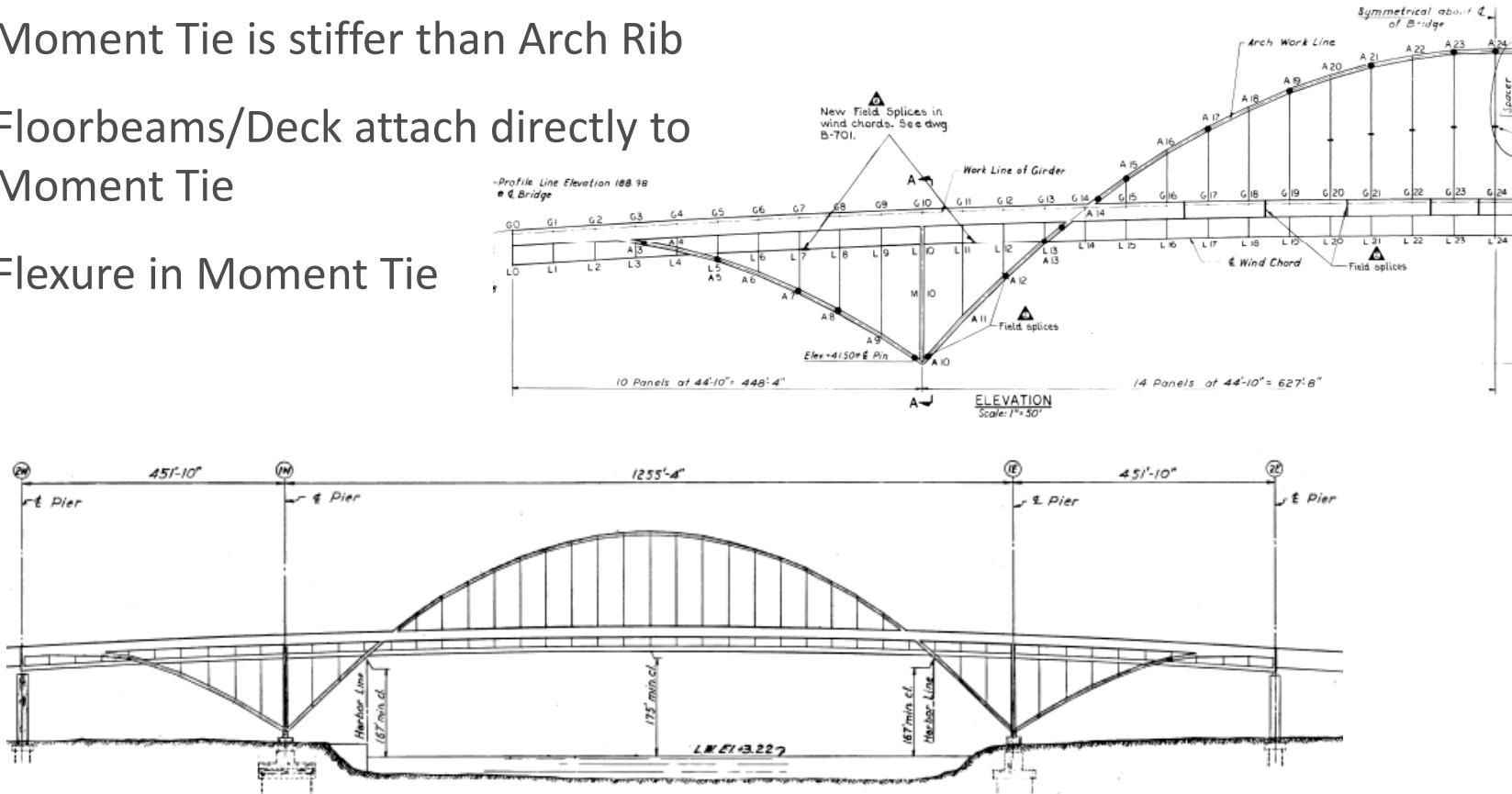


Boundary Conditions

- Moment Tied Arch

- Moment Tie is stiffer than Arch Rib
- Floorbeams/Deck attach directly to Moment Tie
- Flexure in Moment Tie

- ❖ Fremont Bridge





R/S Ratio



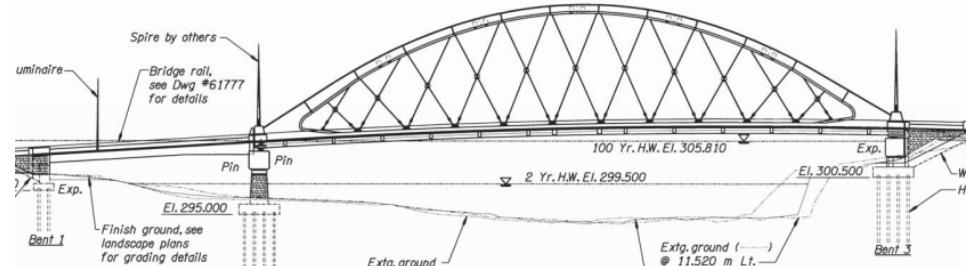
Constant Section



Boundary Conditions

- Network Tied Arch
 - Concrete Through Arch
 - Moment Tie
 - Tensioned Hanger Cables

- ❖ Rogue River, Depoe Street Bridge
- ❖ Alsea Bay Bridge



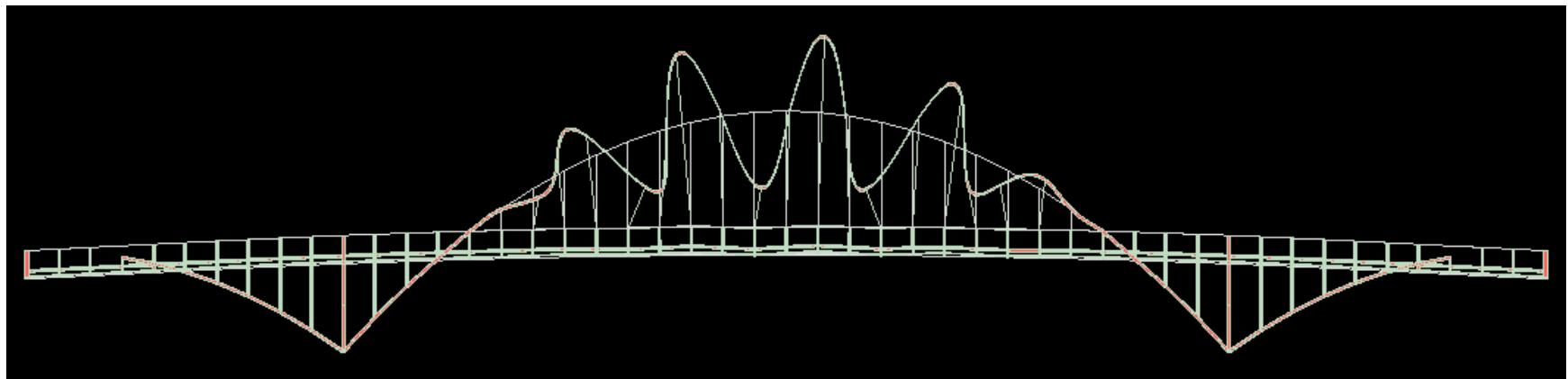
Advanced Methods for Arch Buckling Capacity

- **Eigenvalue Buckling**
- **Nonlinear Pushover Analysis**
- **Other advanced methods**

Eigenvalue Solution

Bifurcation Buckling

$$EI \frac{d^4 w}{dx^4} + P \frac{d^2 w}{dx^2} = 0$$



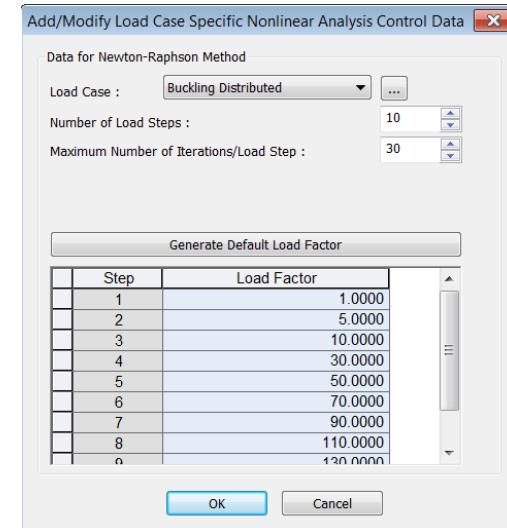
- Pure mathematical solution based on idealized “perfect” condition.
- Oversimplification of buckling behavior can lead to over/under-estimation of critical buckling capacity.
- Useful for determining mode shapes and “ballpark” buckling capacity.

Nonlinear Pushover Analysis

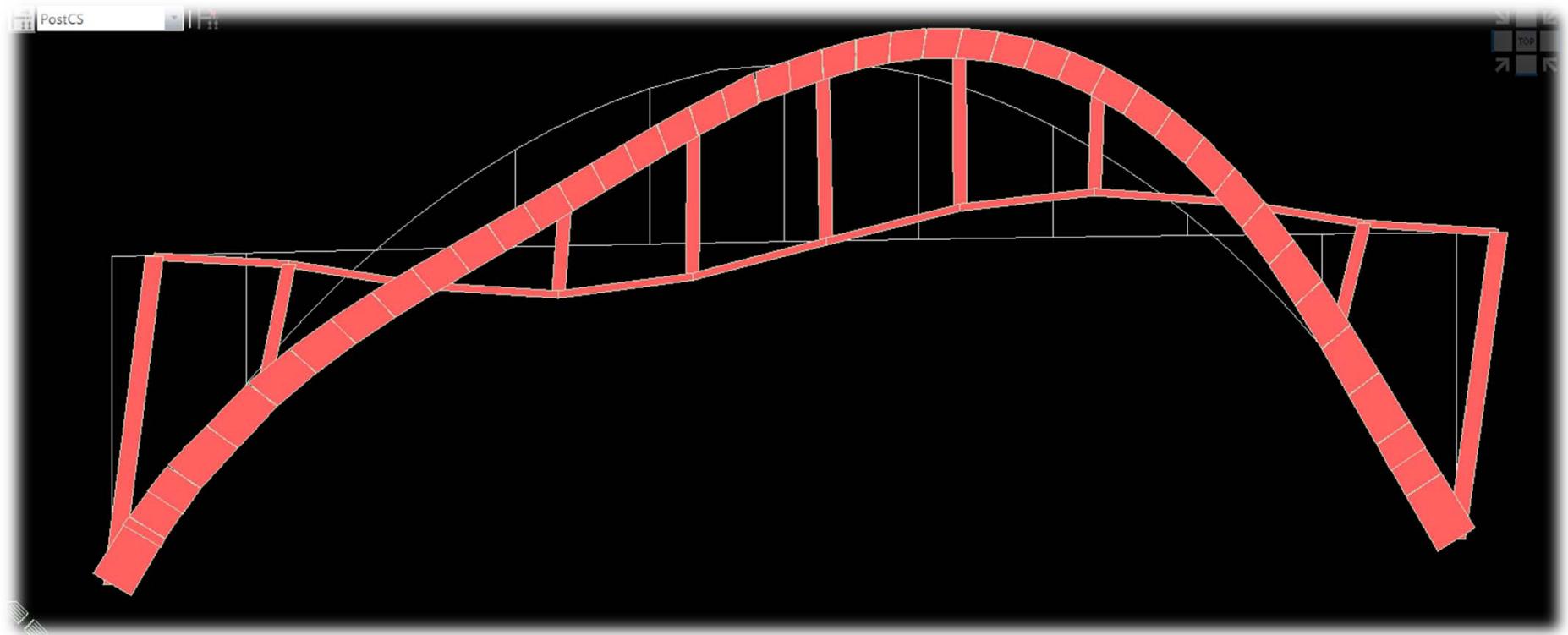
- Provide arch nodes with an initial imperfection in the shape of the buckling mode.
- Apply a uniform distributed load to the bridge deck.
- Use Structural Analysis Software to gradually ramp up Load Factors.
- Observe the structural response.
- The “buckling load” is taken as the axial force at the quarter-point of the rib for the step immediately before non-real results are obtained.

“Just before buckling, small changes in force will yield rapid deformations.”

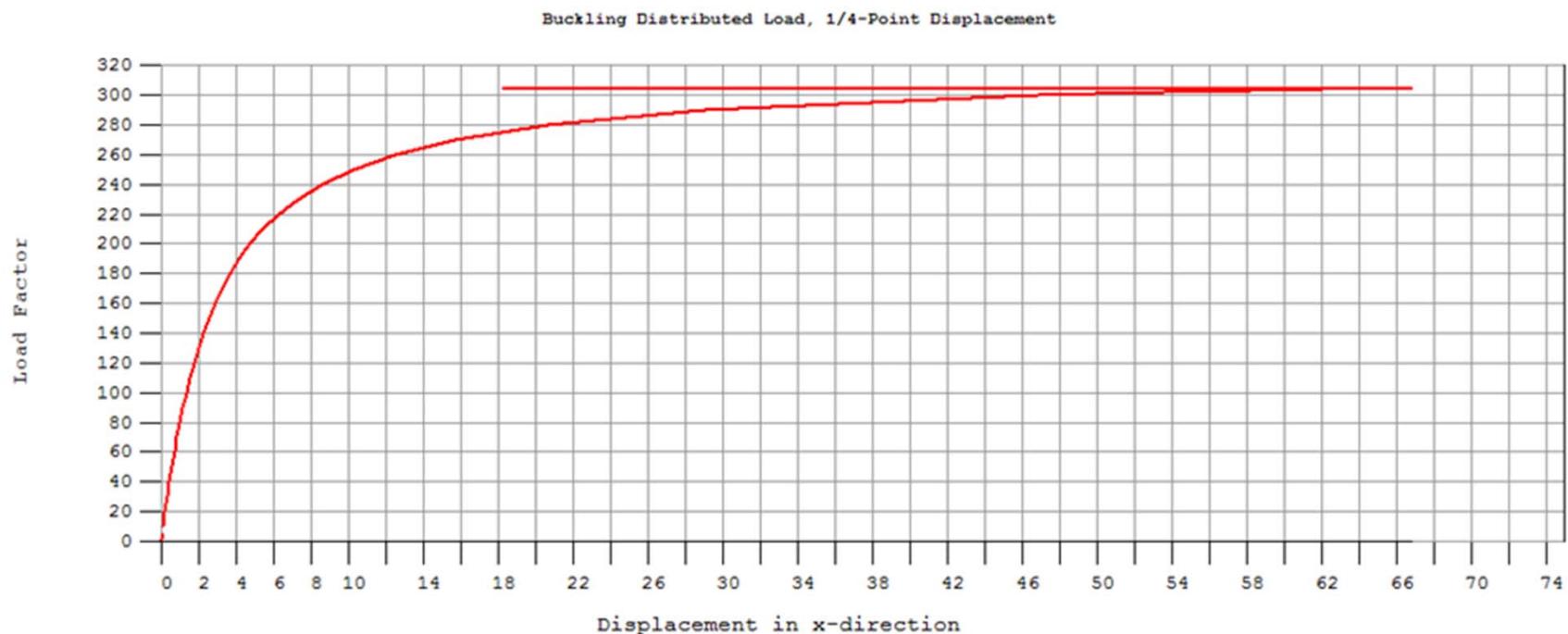
LARSA 4D Documentation: Analysis Reference



Anti-symmetric Sinusoidal 1st Mode

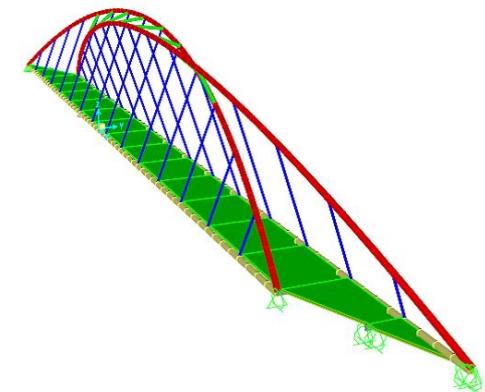
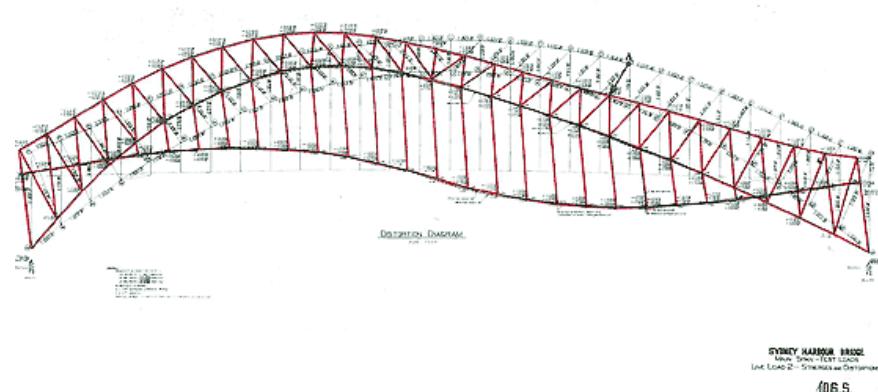
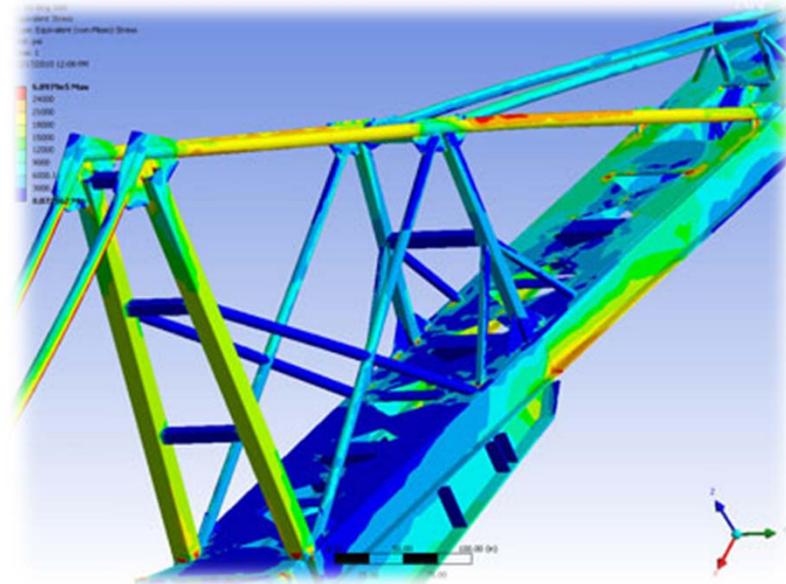


Observation of Failure



“Collapse occurs when the stiffness matrix of the structure becomes singular and thus unsolvable. A singular matrix indicates the structure has no predictable deformation under any loading.” *LARSA 4D Documentation: Analysis Reference*

Other Advanced Methods (FEA and automated Pushover)



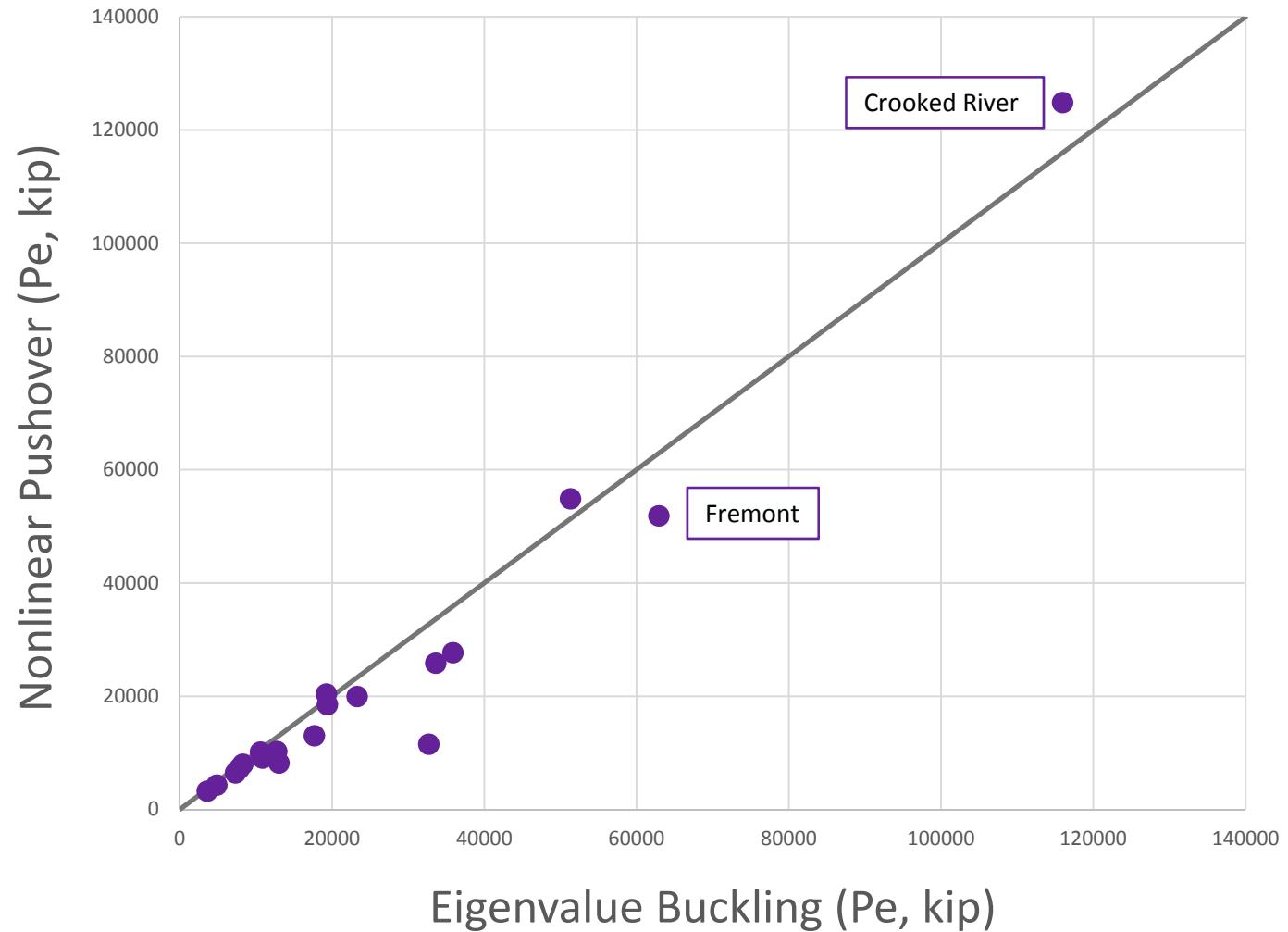
- Labor intensive models, maybe for design?
- Results from automated tools don't correlate well on bridges with complex geometry.

Comparison of Results computed by Arch Buckling Methods

2014 ODOT Arch Bridge Load Rating

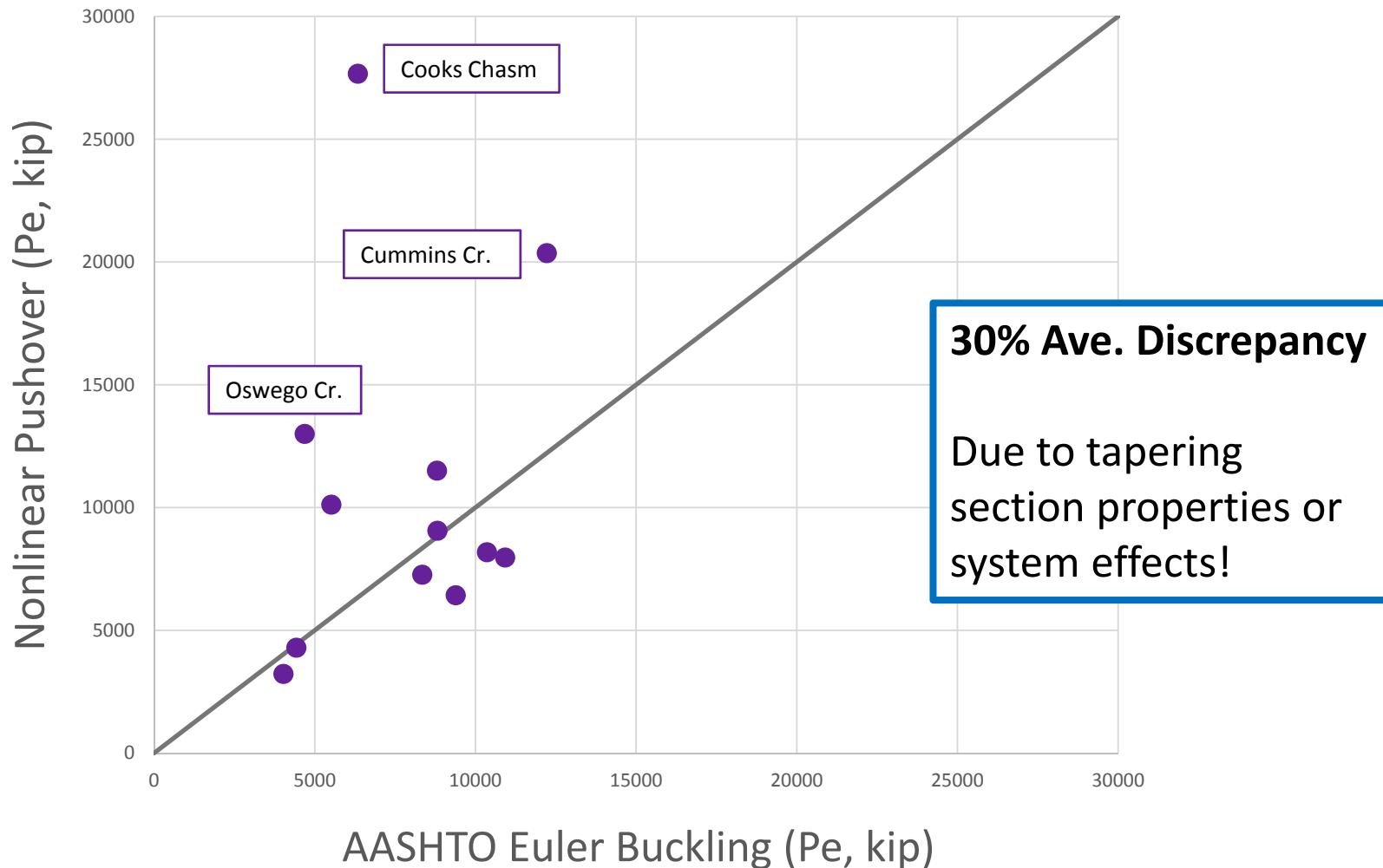
ODOT Bridge #	Name	Arch Boundary Condition	R/S
01182	Cummins Creek, Hwy 25 at MP 28.29	Fixed	0.21
18544	Haynes Inlet Slough, Hwy 9	Two Hinge	0.16
09691	John Day River, Hwy 291	Tied	0.20
02529	Fremont, Hwy 61	Moment Tied	0.27
00231A	Rickreall Creek, Hwy 191 SB	Fixed	0.16
00409	Oswego Creek, Hwy 3 SB	Fixed	0.34
01089	Rocky Creek, Hwy 9 Frtg Rd	Fixed	0.28
01617	Clackamas River, Hwy 1E	Tied	0.21
02459	Depoe Bay, Hwy 9	Fixed	0.27
04534	Multnomah Creek, Hwy 100	Filled Spandrel Wall	0.15
07347	Little North Fork Santiam River, Hwy 162	Fixed	0.19
18096	Brush Creek, Hwy 9 at MP 306.35	Fixed	0.28
18211	Crooked River Gorge, Hwy 4	Fixed	0.19
18960	Cooks Chasm, Hwy 9	Fixed	0.20
19273	Rogue River, Depot St	Network Tied	0.21
20672	Dry Canyon Creek, NW Maple (Negus)	Fix-Pin (3 Arches)	0.29
01418	Rogue River, Hwy 25 SB (6th St, Caveman)	Fixed	0.40
00498	Mosier Creek, Hwy 100	Fixed	0.29
00524	Dry Canyon Creek, Hwy 100	Fixed	0.29

Nonlinear Pushover vs. Eigenvalue Buckling

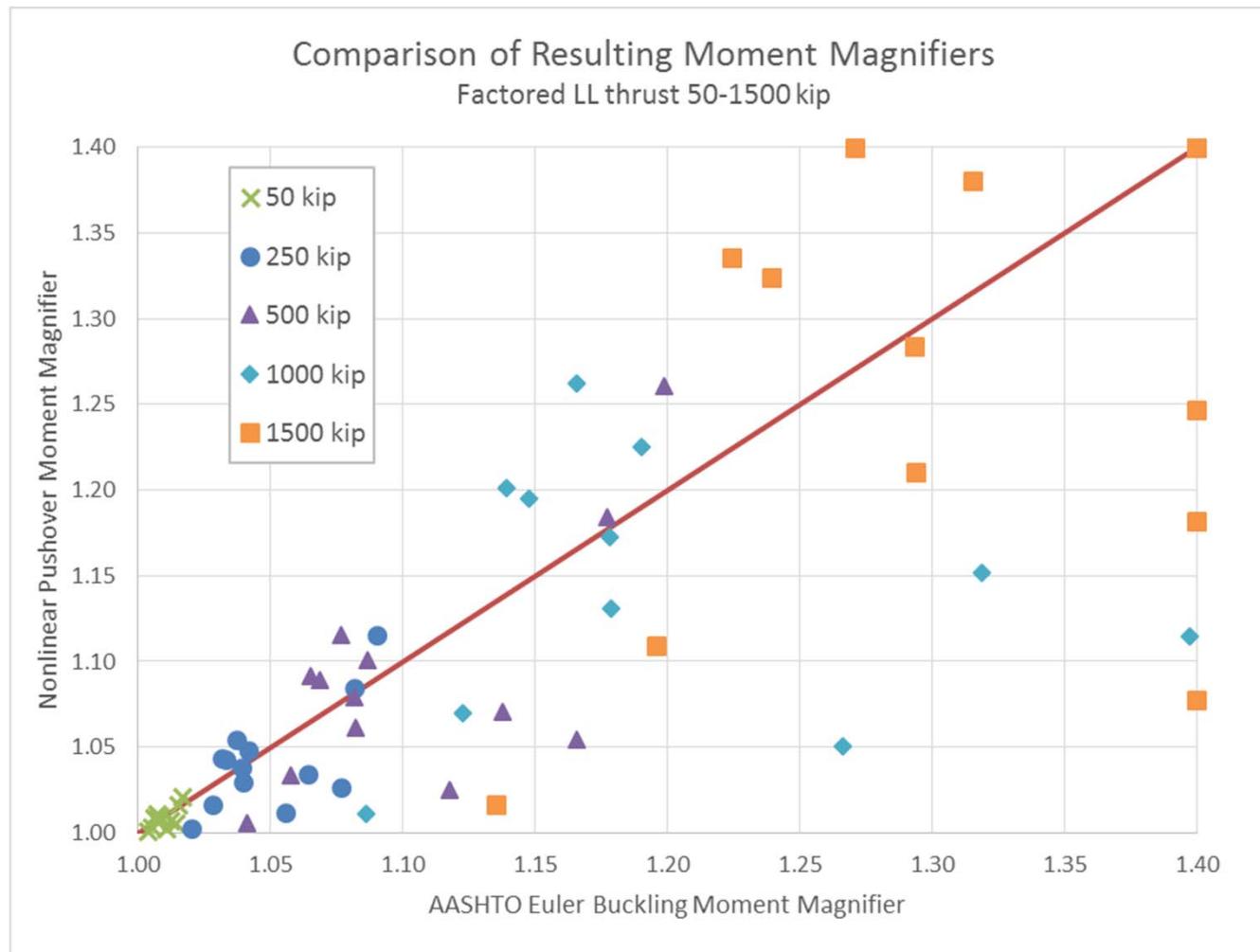


Nonlinear Pushover vs. AASHTO Euler Buckling

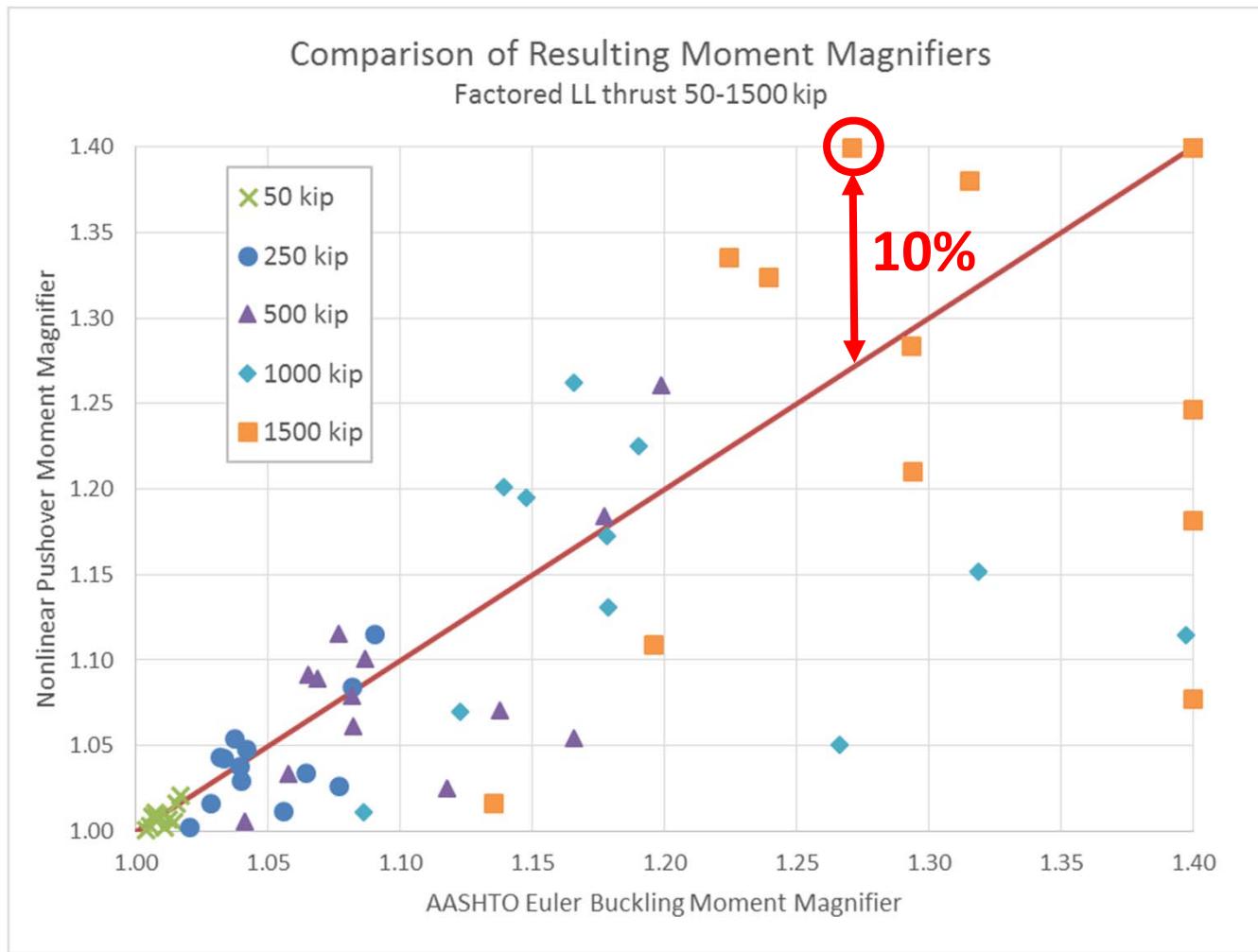
Only comparing arches with R/S and B.C. that are included in the AASHTO "k" table.



Comparison of Resulting Moment Magnifiers

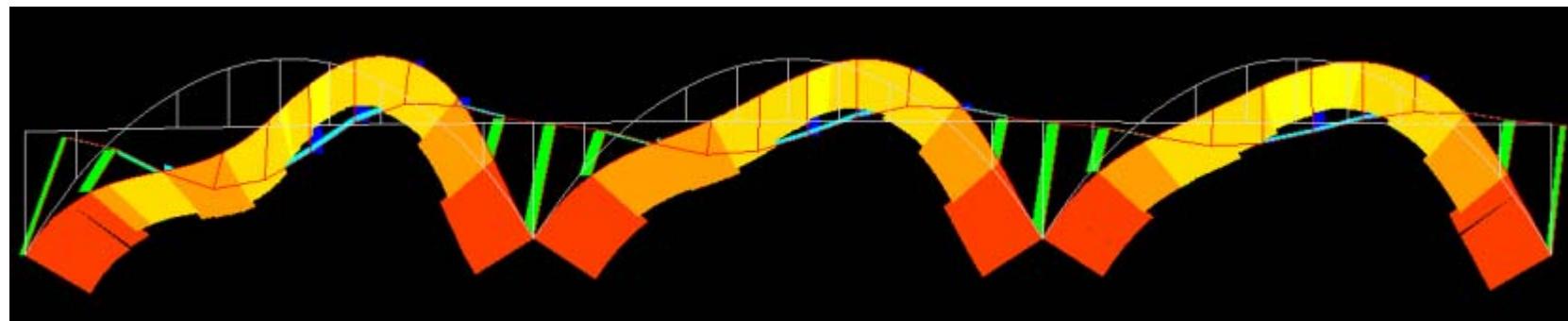


Moment Magnifier from AASHTO Method too low!



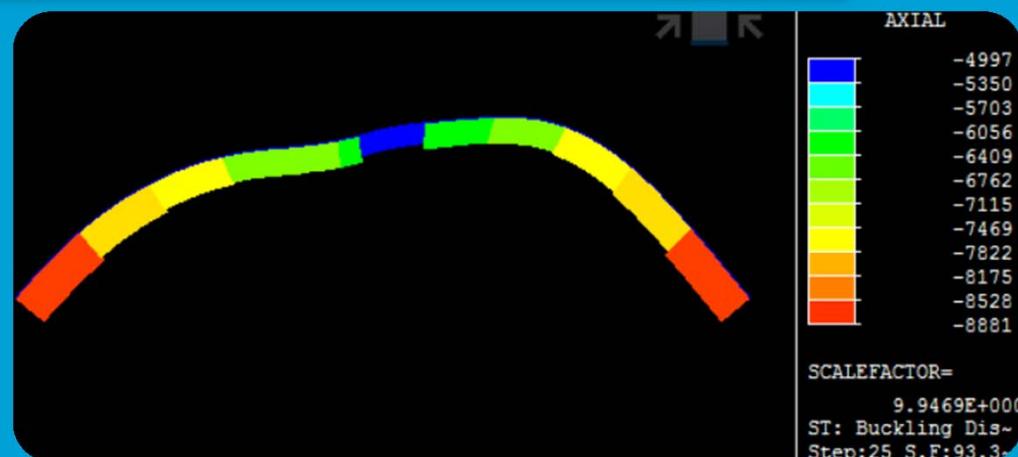
Conclusions

- The guidance provided in AASHTO for calculating arch critical buckling capacity and resulting moment magnifiers is limited.
 - R/S ratio, boundary conditions, constant section properties.
- An Eigenvalue Buckling analysis may overestimate or underestimate the actual buckling capacity (~15%), a Nonlinear Pushover analysis is a more accurate option.
- **If AASHTO “k” values are used for an arch which doesn’t meet the criteria, the resulting moment magnifiers may be lower than they should be!**



Thank You

Questions?



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