



# Live Load Distribution Factor for One- and Two-Cells Box Girders



Presented by:  
Paul Chung P.E.,  
Anthony Logus P.E.,

Structural Analysis Committee  
Caltrans Division of Engineering Services

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# Background

## AASHTO LRFD Chapter 4- Live Load Distribution Factor for Box Girders- for MOMENT (interior web used)

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Distribution Factors	Range of Applicability
Cast-in-Place Concrete Multicell Box	d	One Design Lane Loaded: $\left(1.75 + \frac{S}{3.6}\right) \left(\frac{1}{L}\right)^{0.35} \left(\frac{1}{N_c}\right)^{0.45}$ Two or More Design Lanes Loaded: $\left(\frac{13}{N_c}\right)^{0.3} \left(\frac{S}{5.8}\right) \left(\frac{1}{L}\right)^{0.25}$	$7.0 \leq S \leq 13.0$ $60 \leq L \leq 240$ $N_c \geq 3$  If $N_c > 8$ use $N_c = 8$
Concrete Deck on Concrete Spread Box Beams	b, c	One Design Lane Loaded: $\left(\frac{S}{3.0}\right)^{0.35} \left(\frac{Sd}{12.0L^2}\right)^{0.25}$ Two or More Design Lanes Loaded: $\left(\frac{S}{6.3}\right)^{0.6} \left(\frac{Sd}{12.0L^2}\right)^{0.125}$	$6.0 \leq S \leq 18.0$ $20 \leq L \leq 140$ $18 \leq d \leq 65$ $N_b \geq 3$
		Use Lever Rule	$S > 18.0$



# Background

## AASHTO LRFD Chapter 4- Live Load Distribution Factor for Box Girders- SHEAR (interior web used)

Type of Superstructure	Applicable Cross-Section from <a href="#">Table 4.6.2.2.1-1</a>	One Design Lane Loaded	Two or More Design Lanes Loaded	Range of Applicability
Cast-in-Place Concrete Multicell Box	$d$	$\left(\frac{S}{9.5}\right)^{0.6} \left(\frac{d}{12.0L}\right)^{0.1}$	$\left(\frac{S}{7.3}\right)^{0.9} \left(\frac{d}{12.0L}\right)^{0.1}$	$6.0 \leq S \leq 13.0$ $20 \leq L \leq 240$ $35 \leq d \leq 110$ $N_c \geq 3$



# Purpose

**In 2008 30% of Box Girder Bridges Built in California were WIDENINGS.**

**DIVISION OF ENGINEERING SERVICES  
BRIDGE SQUARE FOOT COST SUMMARY  
2008**

Bridge Code	BRIDGE TYPE	TOTAL NUMBER OF BRIDGES	NUMBER OF BRIDGE WIDENED	AMOUNT**	SQ.FT. OF DECK	AVERAGE COST/SQ.FT.**
10	RC SLAB	10	6	\$8,868,728	47,026	\$189
20	RC T-BEAM	0	0	\$0	0	\$0
21	RC U GIRDER	0	0	\$0	0	\$0
22	RC BOX GIRDER	7	4	\$17,947,370	107,045	\$168
30	CIP/PS U GIRDER	0	0	\$0	0	\$0
31	CIP/PS BOX GIRDER	57	16	\$224,873,282	1,353,837	\$166
32	CIP/PS SLAB	3	0	\$6,436,659	21,850	\$295
40	PC/PS SLAB	0	0	\$0	0	\$0
41	PC/PS "I" GIRDER	4	2	\$10,116,584	44,118	\$229
42	PC/PS "T" GIRDER	0	0	\$0	0	\$0
43	PC/PS "INV T" GIRDER	0	0	\$0	0	\$0
44	PC/PS BOX GIRDER	0	0	\$0	0	\$0
45	PC/PS BULB "T" GIRDER	2	0	\$3,952,372	16,698	\$237
46	PC/PS BOX GIRDER-SEG	0	0	\$0	0	\$0
50	STEEL GIRDER	0	0	\$0	0	\$0
	<b>TOTALS</b>	<b>83</b>	<b>28</b>	<b>\$272,194,994</b>	<b>1,590,574</b>	<b>\$171</b>



# Objectives

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- Use of approximate methods on less-typical structures is prohibited.
- 3D finite element analysis (FEA) must be used.
  - ✓ One or two-cell box girders;
  - ✓ Two or three-girder beam-slab structures;
  - ✓ Spans greater than 240-ft in length;
  - ✓ Structures with extra-wide overhangs (greater than one-half of the girder spacing or 3 ft).
- Use a comprehensive parametric study using 3D FE models.
- Provide new set of simplified equations for computing Live Load Distribution Factors of one- or two-cell box girders



## Scope Of Study

Superstructure edge to edge width:  $W$

Structure depth:  $d$

Span length:  $L$

Girder spacing:  $S$

Number of spans:  $N_s$

Table 2.1: Range of study parameters for  $LDF$

$W$	12 – 24 ft (1-Cell) 12 – 36 ft (2-Cell)
$d$	4 – 9 ft
$L$	65 – 220 ft
$d/L$	0.04 – 0.06
$S$	6, 7, 8, 9, 10, 13 ft
$S/d$	1 – 2



## Parameter Matrix

Table 2.2: Study Matrix for One-Cell Box Girder

#	Bridge Deck Width, $W$ (ft)	Span Length, $L$ (ft)	Structure Depth, $d$ (ft)	$d/L$ Ratio	Spacing, $S$ (ft)	$S/d$ Ratio
1	12	65	3.9	0.06	6	1.54
2	12	100	4.0	0.04	6	1.50
3	12	100	5.0	0.05	6	1.20
4	20	100	5.0	0.05	10	2.00
4-1	14	100	5.0	0.05	7	1.4
4-2	16	100	5.0	0.05	8	1.6
4-3	18	100	5.0	0.05	9	1.80
5	12	100	6.0	0.06	6	1.00
6	20	100	6.0	0.06	10	1.67
6-1	18	100	6.0	0.06	9	1.50
7	23	100	6.0	0.06	13	2.17
8	12	150	6.0	0.04	6	1.00
9	20	150	6.0	0.04	10	1.67
9-1	18	150	6.0	0.04	9	1.50
10	23	150	6.0	0.04	13	2.17
11	20	180	8.0	0.044	10	1.25
11-1	18	180	8.0	0.044	9	1.125
12	23	180	8.0	0.044	13	1.63
13	20	220	9.17	0.042	10	1.09
13-1	18	220	9.17	0.042	9	.981
14	23	220	9.17	0.042	13	1.42



## Parameter Matrix

Table 2.3: Study Matrix for Two-Cell Box Girder

#	Bridge Deck Width, $W$ (ft)	Span Length, $L$ (ft)	Structure Depth, $d$ (ft)	$d/L$ Ratio	Spacing, $S$ (ft)	$S/d$ Ratio
1	18	65	3.9	0.06	6	1.54
2	18	100	4	0.04	6	1.50
2-1	24	75	4	0.04	8	2.00
3	18	100	5	0.05	6	1.20
4	30	100	5	0.05	10	2.00
4-1	27	100	5	0.05	9	1.80
5	18	100	6	0.06	6	1.00
6	30	100	6	0.06	10	1.67
6-1	27	100	6	0.06	9	1.50
7	36	100	6	0.06	13	2.17
7-1	36	100	6	0.06	13	2.17
8	18	150	6	0.04	6	1.00
9	30	150	6	0.04	10	1.67
9-1	27	150	6	0.04	9	1.50
10	36	150	6	0.04	13	2.17
11	30	180	8	0.044	10	1.25
11-1	27	180	8	0.044	9	1.125
12	36	180	8	0.044	13	1.63
13	30	220	9.17	0.042	10	1.09
13-1	27	220	9.17	0.042	9	.981
14	36	220	9.17	0.042	13	1.42





# Parameter Matrix

## Selected Cases for two and three span bridges

#	Bridge Deck Width, $W$ (ft)	Spans (ft)	Structure Depth, $d$ (ft)	$d/L$ Ratio	Spacing, $S$ (ft)	$S/d$ Ratio
Case 1	36	100	6	0.06	13	2.17
Case 2	36	100,100	6	0.06	13	2.17
Case 3	36	100,140,100	6	0.043	13	2.17
Case 4	36	100	4.5	0.045	13	2.89
Case 5	36	100,100	4.5	0.045	13	2.89
Case 6	36	100,140,100	4.5	0.032	13	2.89
Case 7	36	75,100,75	4.5	0.045	13	2.89
Case 8	36	165,220,165	9	0.041	13	1.44
Case 9	24	100	6	0.06	8	1.33
Case 10	24	100,100	6	0.06	8	1.33
Case 11	24	100,140,100	6	0.043	8	1.33
Case 12	24	100	4.5	0.045	8	1.78
Case 13	24	100,100	4.5	0.045	8	1.78
Case 14	24	100,140,100	4.5	0.032	8	1.78
Case 15	24	75,100,75	4.5	0.045	8	1.78
Case 16	24	165,220,165	9	0.041	13	1.44



# Analysis

## Trucks Types Used for Study

### 2.3 LIVE LOADS

- HL-93K: design tandem and design lane load;
- HL-93M: design truck and design lane load;
- HL-93S: 90% of two design trucks and 90% of the design lane load;
- HL-93LB: pair of one design tandem and one design lane load.

Table 2.5: Multiple presence factor (MPF), notice it also applies to fractional number of lanes

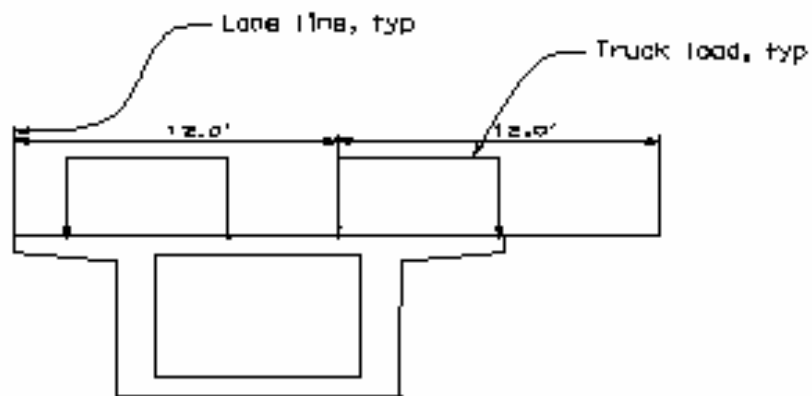
Number of Loaded Lanes	Multiple Presence Factor
$\leq 1$	1.20
$>1$ and $\leq 2$	1
$>2$ and $\leq 3$	0.85

# Analysis

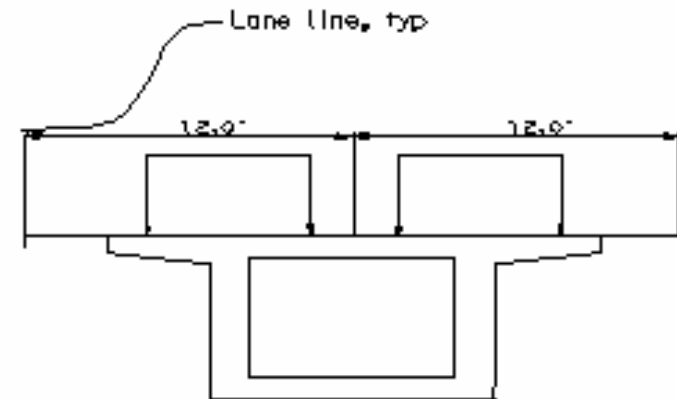
## Live Load Lane Configuration- Transverse Direction

b)  $1 < \# \text{ Lanes} < 2$

#	Cell Type	L (ft)	D (ft)
1	1B5	100	5



X1B  $t_d = 8.125'$   
 $t_g = 7.5'$   
 $t_g = 12.0'$



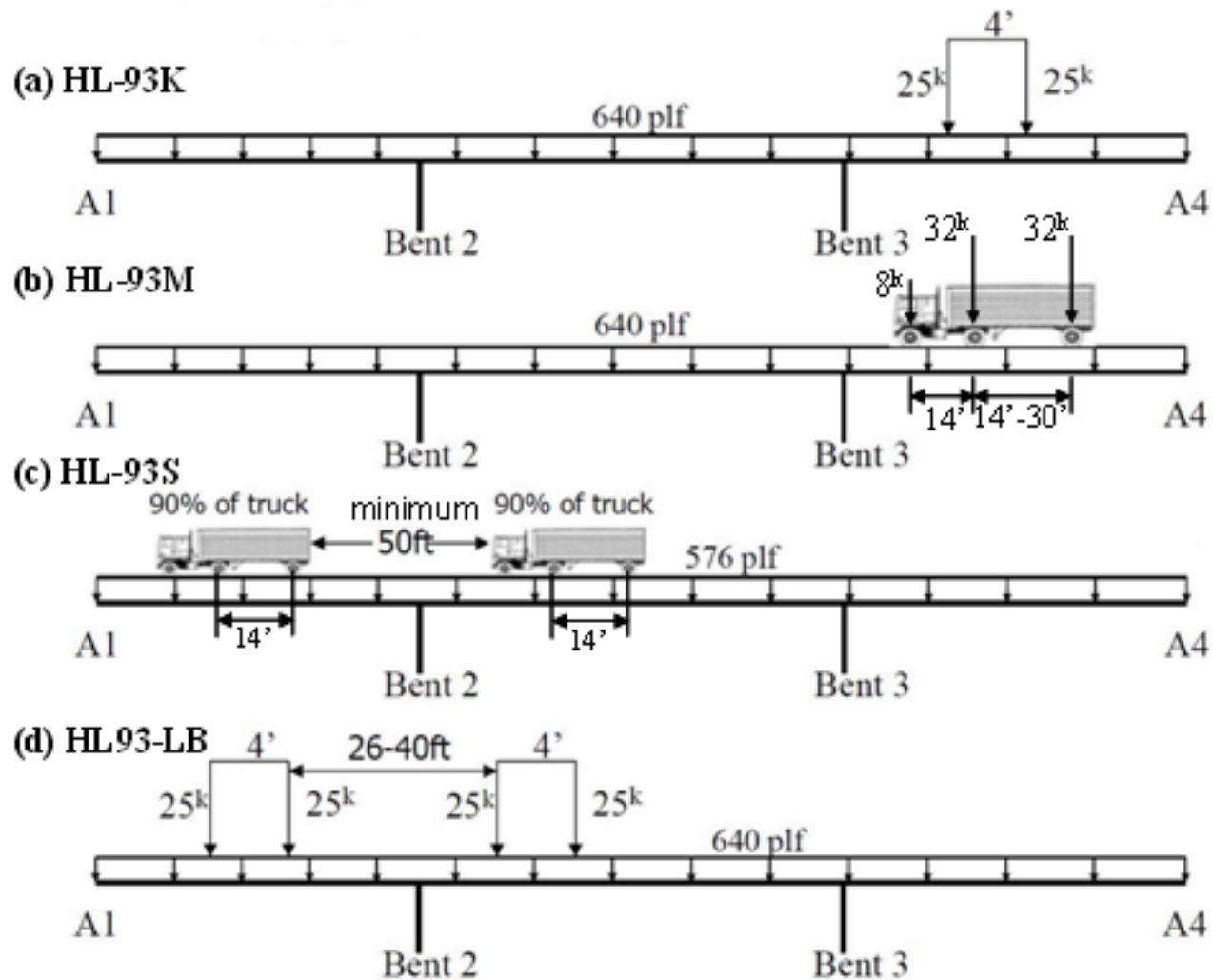
X1B  $t_d = 8.125'$   
 $t_g = 7.5'$   
 $t_g = 12.0'$

A.1.3 Girder Spacing = 8', Maximum Number of Lanes < 2

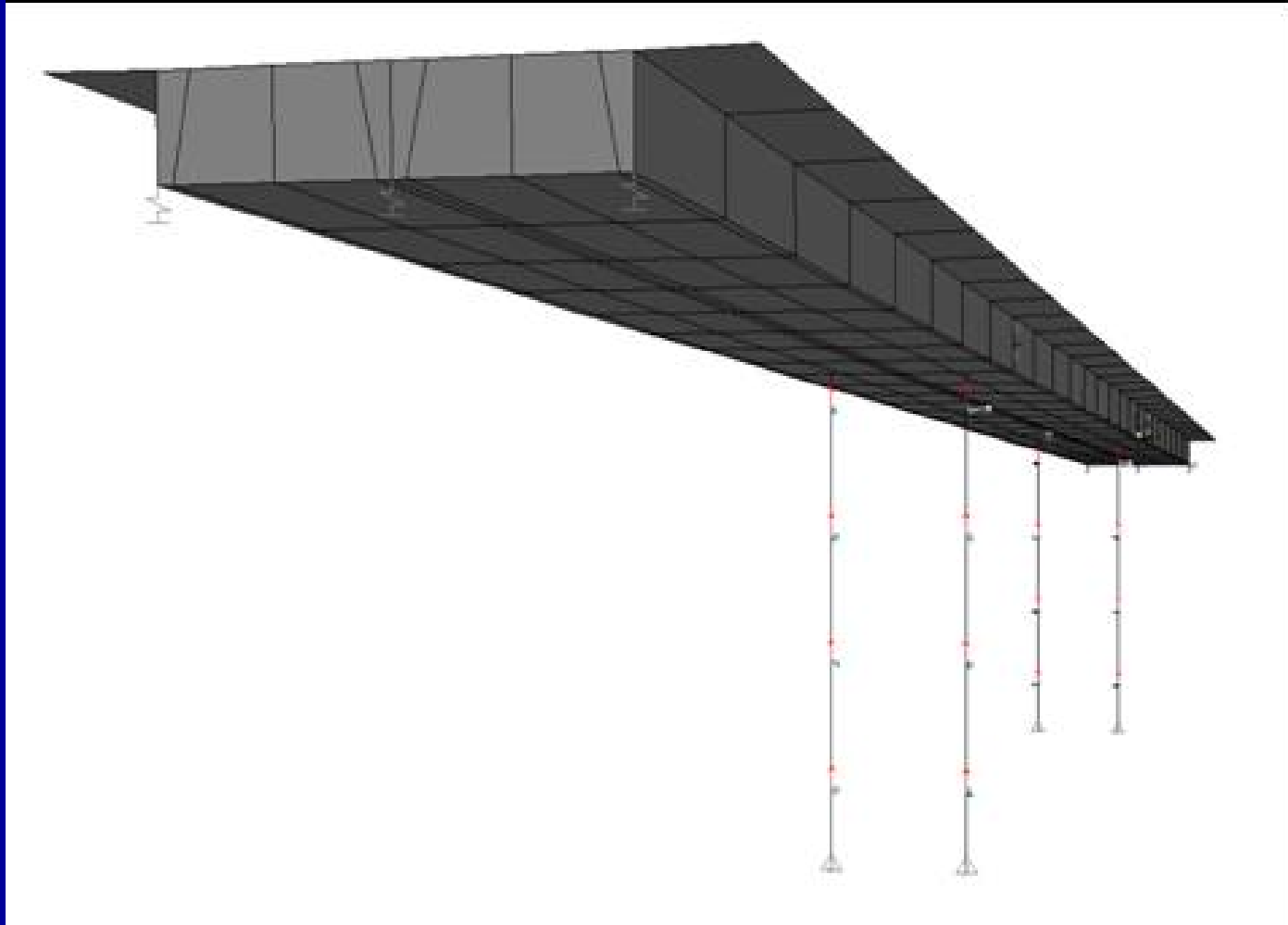


# Analysis

## Live Load Lane Configuration- Longitudinal Direction



# *SAP2000 3-D FEA Model*



# Analysis Results and Evaluation

Part 1: Compare SAP2000 LDF vs AASHTO LRFD equations

- Live-load distribution factor (LDF) for individual girders
- Use the maximum force from selected girder

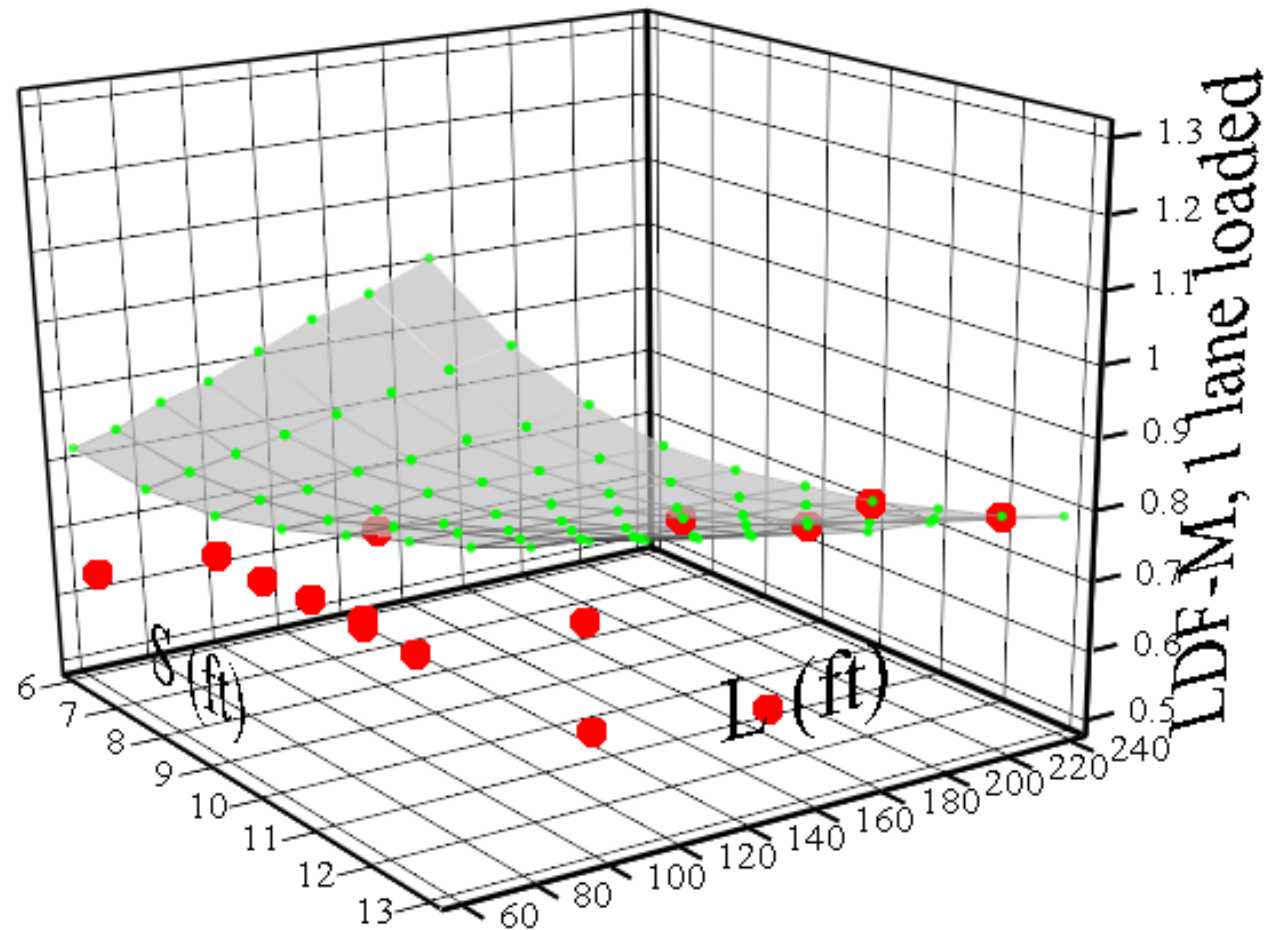
Part 2: SAP2000 TLL and Curve-Fitting Equation

- Total Live Load Lanes (TLL) for the given bridge deck
- $TLL = \text{Maximum force} \times \text{Numbers of girders}$
- Conservative for design
- Curve-fitting: Find relevant parameters and suitable empirical equations



# Analysis Results vs AASHTO Equations

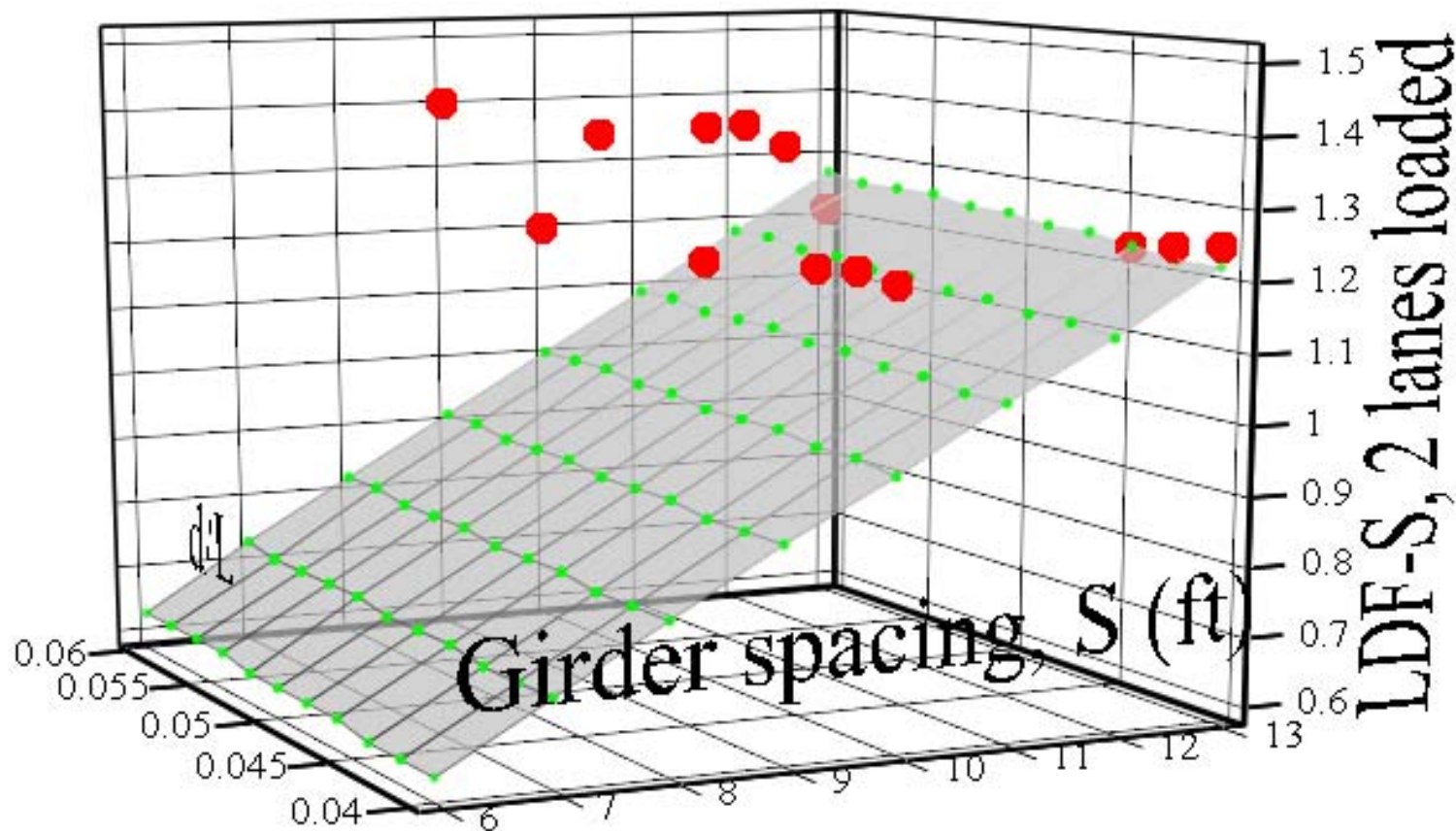
Figure 1m: Moment LDF (individual girder)- One-Cell box, one lane loaded, surface: AASHTO LDF equation; scatter points: LDF from SAP2000





# Analysis Results vs AASHTO Equations

Figure 1s: **Shear LDF (individual girder) One-Cell box, two lanes loaded,**  
surface: AASHTO LDF equation;  
scatter points: LDF from SAP2000

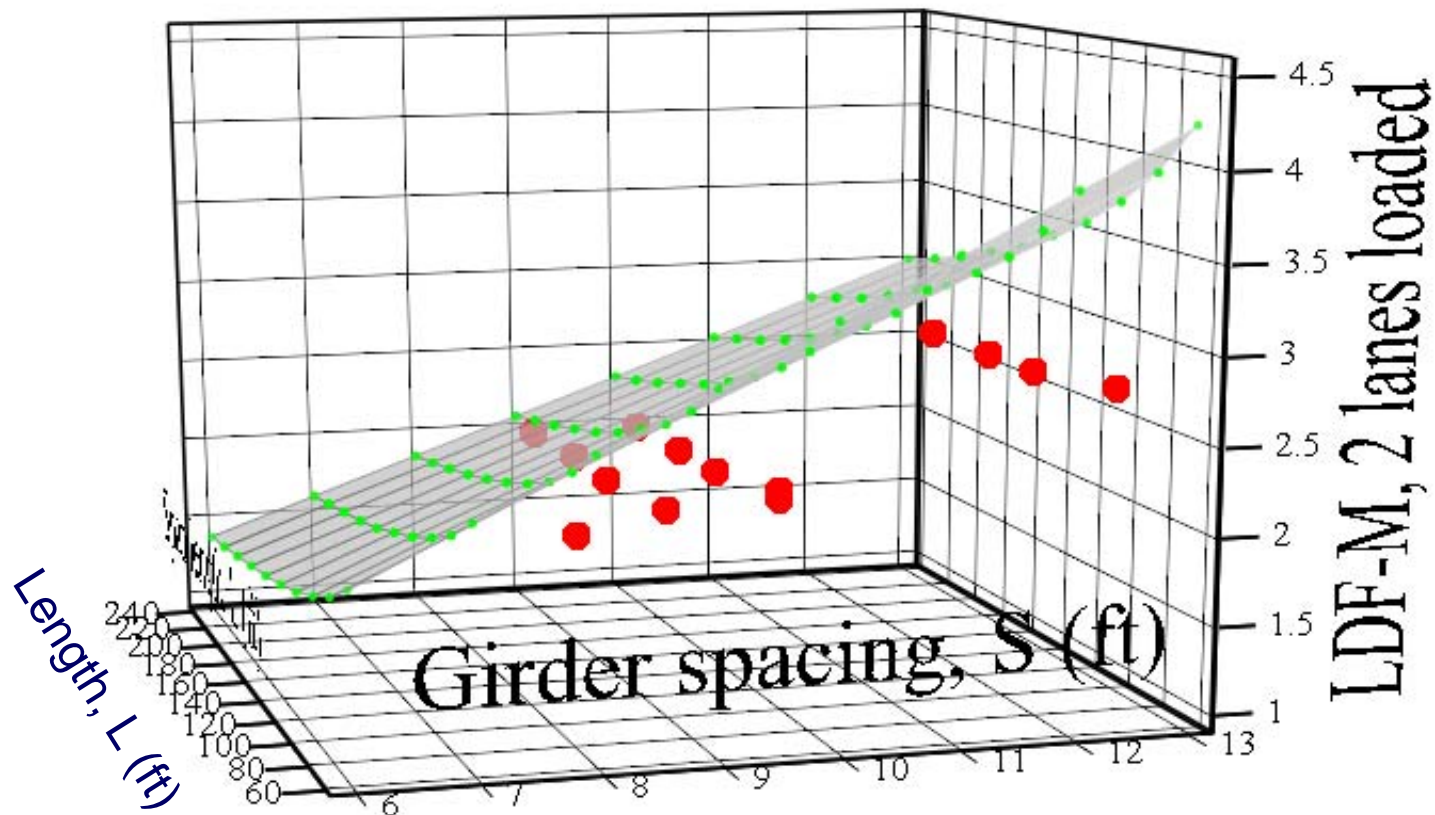






# Analysis Results vs. AASHTO Equations

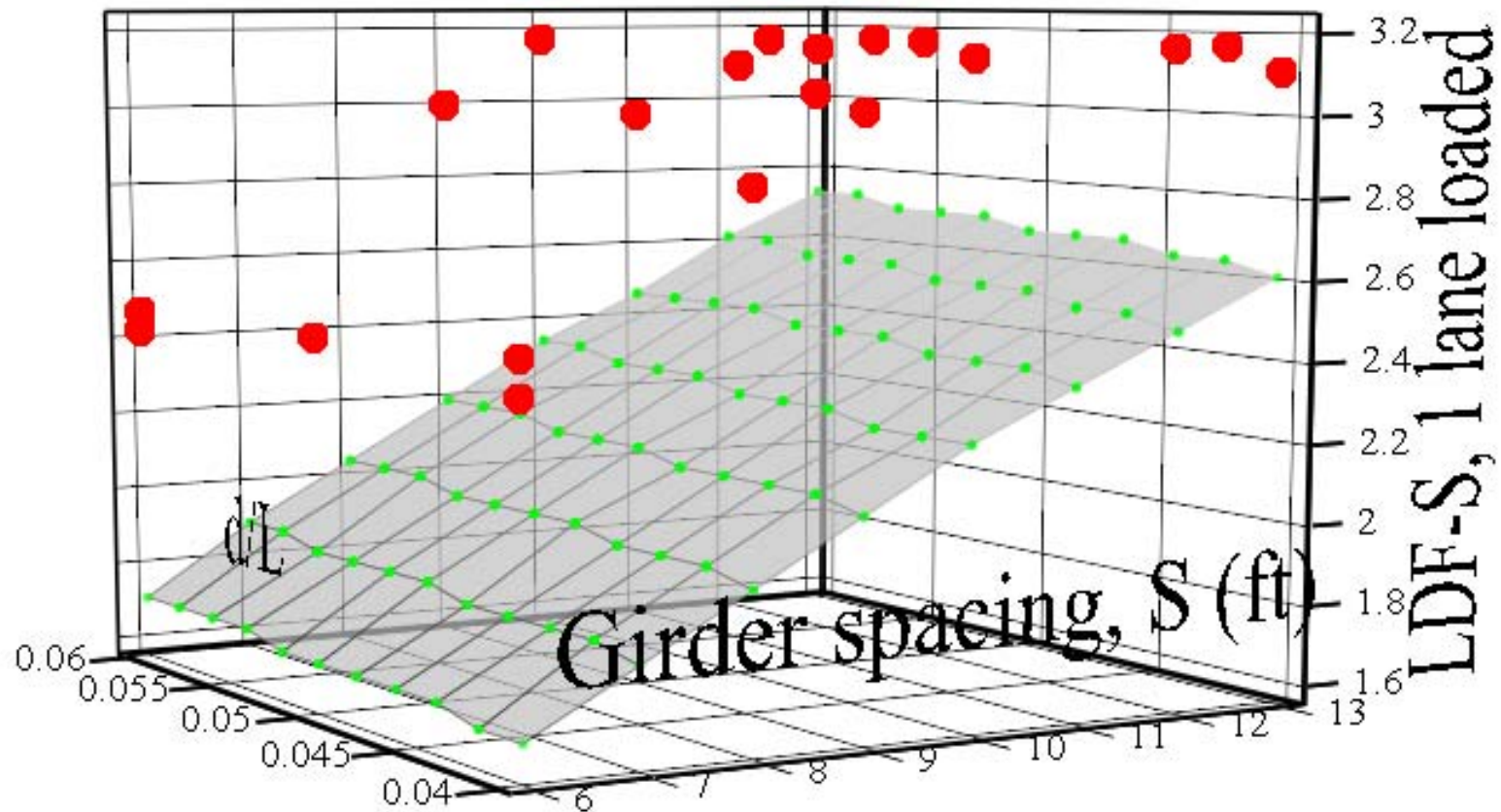
Figure 2m: **Moment LDF (individual girder)- Two-Cell box, Two lanes loaded,**  
surface: AASHTO LDF equation;  
scatter points: LDF from SAP2000





# Analysis Results vs AASHTO Equations

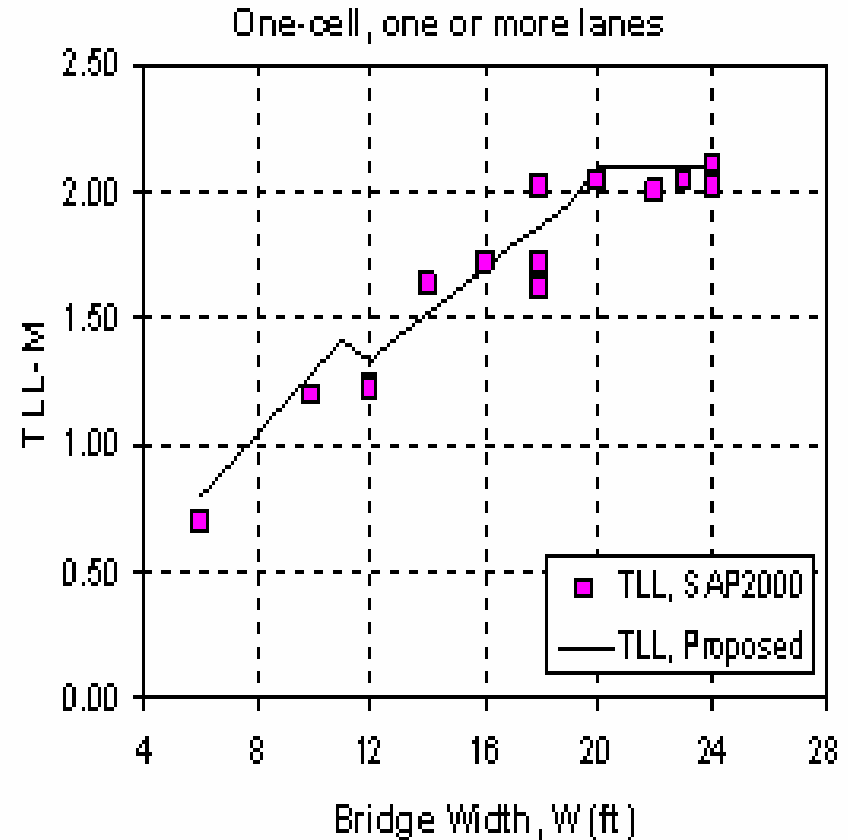
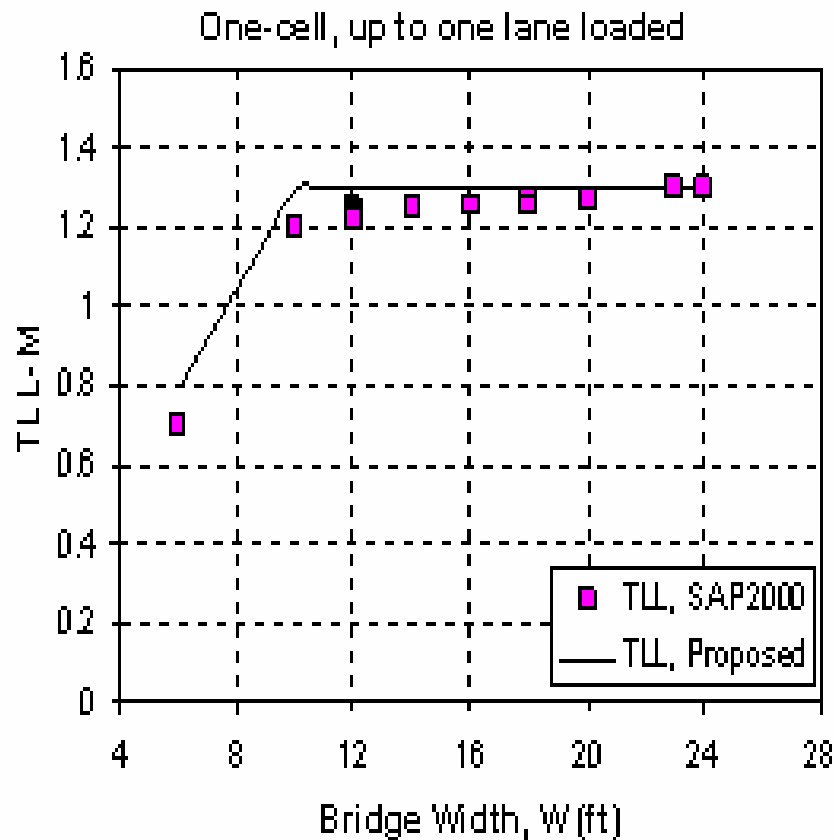
Figure 2s: Shear LDF (individual girder)- Two-Cell box, one lane loaded, surface: AASHTO LDF equation; scatter points: LDF from SAP2000





## TLL Analysis Results- Moment

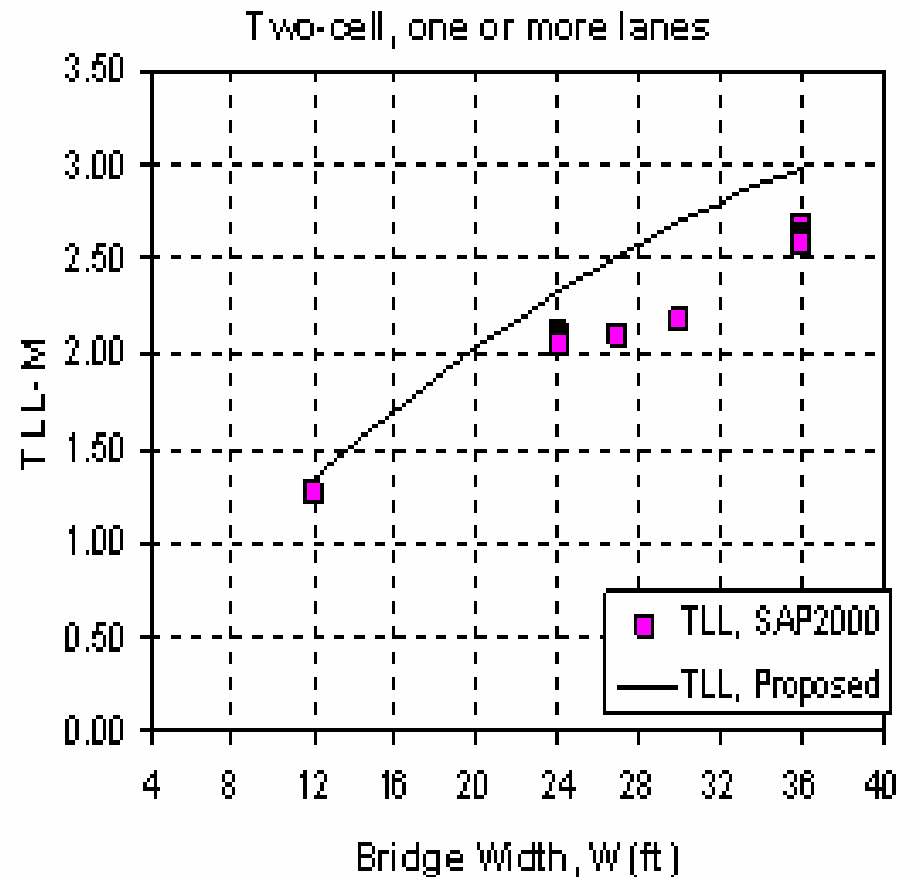
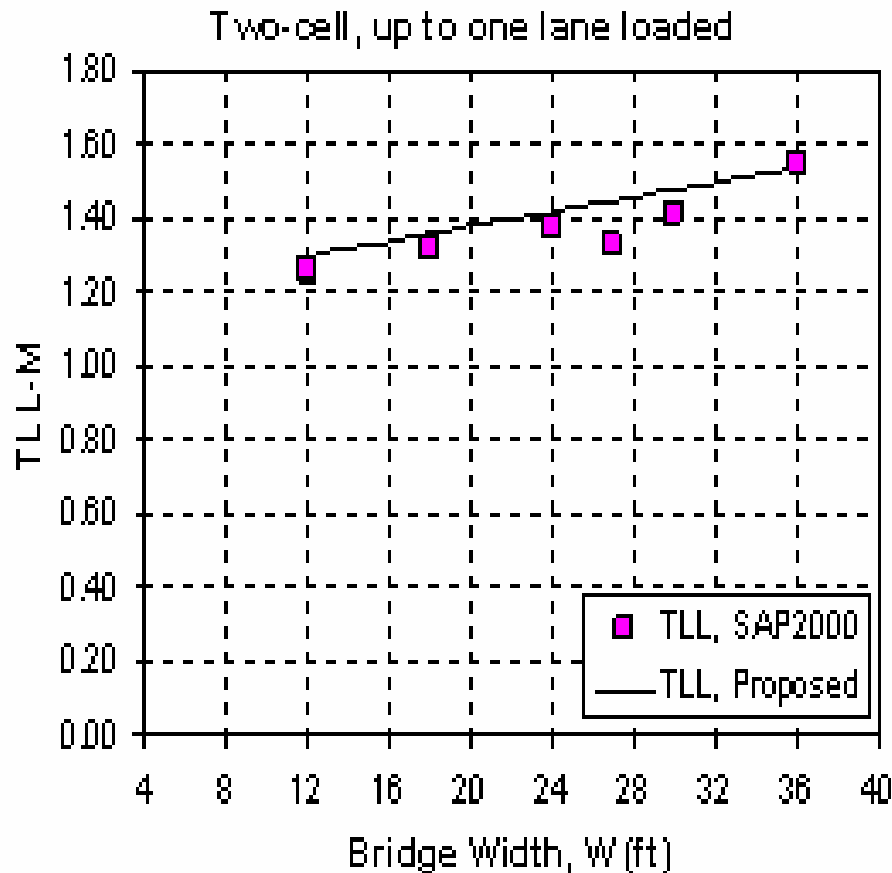
Figure 4m: TLL computed from SAP2000, **One-Cell**;  
TLL depended on Bridge Widths, W





# TLL Analysis Results- Moment

Figure 5m: TLL computed from SAP2000, Two-Cell;  
TLL depended on Bridge Widths, W





## TLL Results- Moment

### 3.3.1 Proposed Equations for Moment TLL

It is found that the moment TLL for one-cell and two-cell box girders depends only on the bridge width,  $W$ . The following equations have been proposed:

for one-cell box girders:

$$\text{up to one lane loaded : } \begin{cases} \frac{W}{12}(1.65 - 0.01W) & 6 \leq W < 10 \\ 1.3 & 10 \leq W \leq 24 \end{cases} \quad (3-3)$$

$$\text{more than one lane loaded : } \begin{cases} \frac{W}{12}(1.65 - 0.01W) & 6 \leq W < 12 \\ \frac{W}{12}(1.5 - 0.014W) & 12 \leq W < 20 \\ 2.1 & 20 \leq W \leq 24 \end{cases} \quad (3-4)$$

for two-cell box girders:

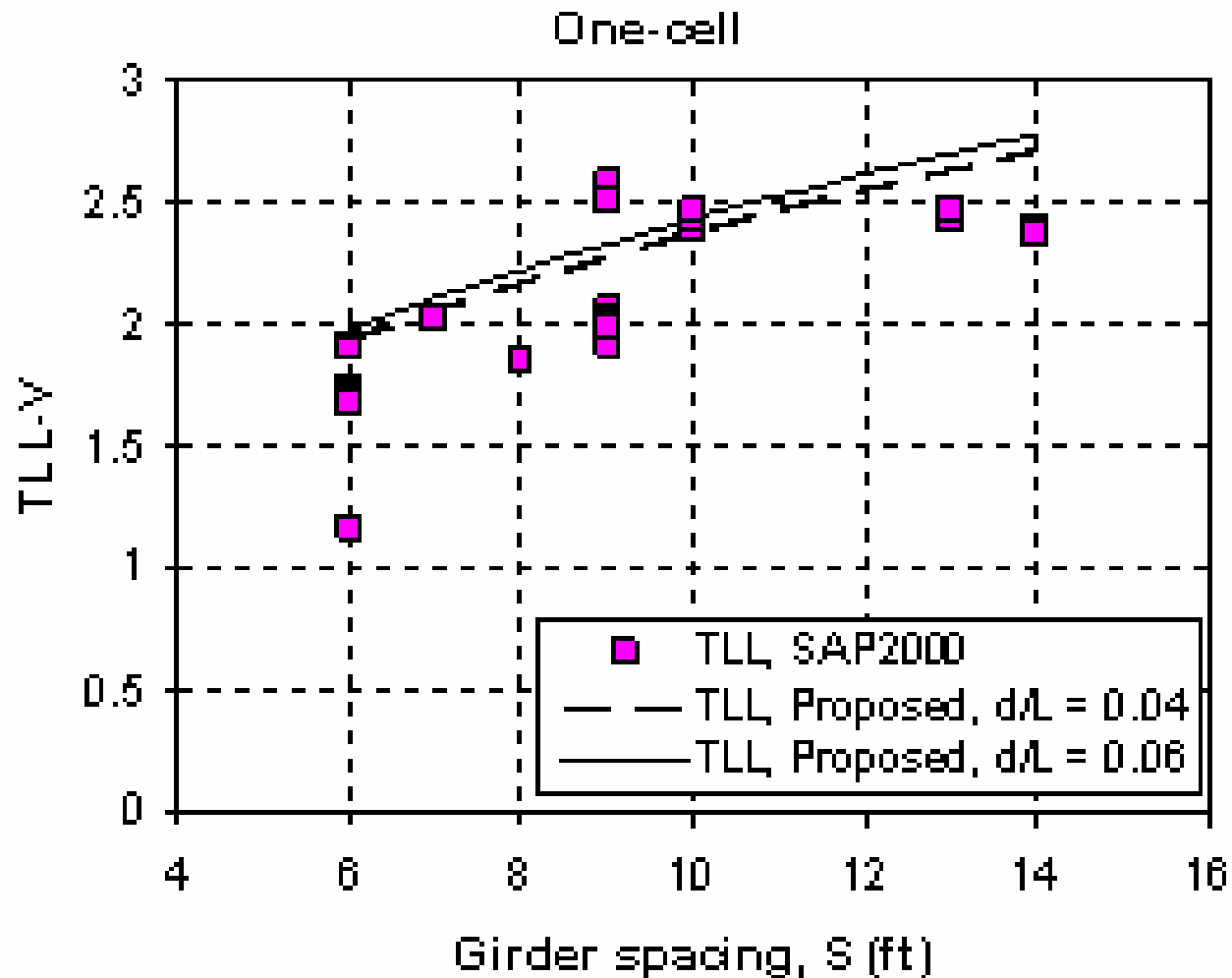
$$\text{up to one lane loaded : } 1.3 + 0.01(W - 12) \quad 12 \leq W \leq 36 \quad (3-5)$$

$$\text{more than one lane loaded : } \frac{W}{12}(1.5 - 0.014W) \quad 12 \leq W < 36 \quad (3-6)$$



# TLL Analysis Results- Shear

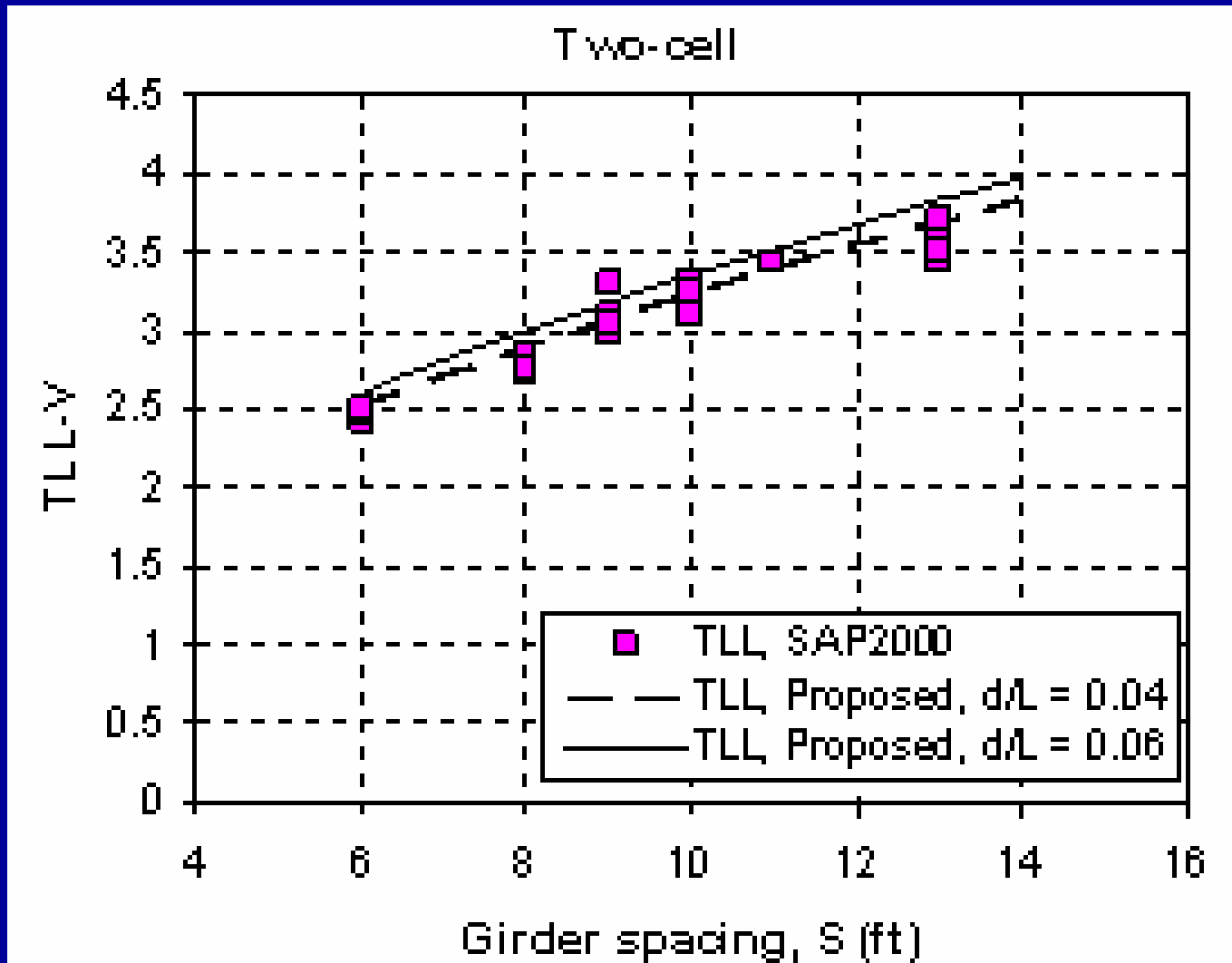
Figure 4s: TLL computed from SAP2000, One-Cell;  
TLL depended on S and d/L ratio





## TLL Analysis Results- Shear

Figure 4s: TLL computed from SAP2000, Two-Cell;  
TLL depended on S and d/L ratio





## *TLL Results- Shear*

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### 3.3.2 Proposed Equations for Shear TLL

It is found that the shear TLL depends primarily on the girder spacing,  $S$ , which is consistent with AASHTO equations. The AASHTO equations for shear LDF also weakly depend on  $d/L$ . In a consistent way, the following equations are proposed for the shear TLL:

for one-cell box girders:

$$2. \left(\frac{S}{4}\right)^{0.4} \left(\frac{d}{12L}\right)^{0.06} \quad (3-7)$$

for two-cell box girders:

$$3. \left(\frac{S}{4}\right)^{0.5} \left(\frac{d}{12L}\right)^{0.09} \quad (3-8)$$





## SAP2000 MODEL VERIFICATION

Table 5.1: LDF comparison between for one-span bridge

$N_c$	$S$	$W$	$L$	$L/W$	Shear		Moment	
					SAP2000	Midas	SAP2000	Midas
2	13	36	75	2.08	1.16	1.28	1.1	1.04
2	7	24	75	3.13	0.96	0.96	0.85	0.77
1	13	23	100	4.34	1.20	1.13	1.02	1.02
1	6	12	65	5.41	0.91	0.88	0.62	0.52
1	13	23	220	9.56	1.24	1.00	1.01	1.00

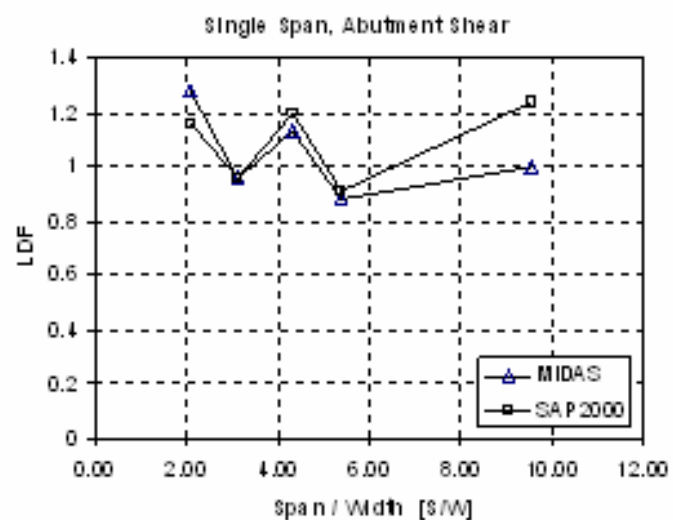
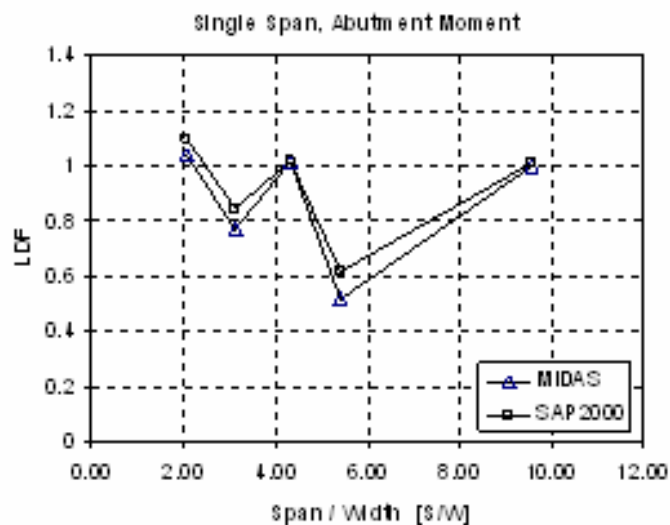


Figure 5.1: Comparison of shear and moment LDFs between Midas Civil 2006 and SAP2000.



# CA Amendment V 4.08.01- Chapter 4

**Table 4.6.2.2.2b-2 Total Design Live Load Lanes for Moment**

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Total Live Load Design Lanes	Range of Applicability
Cast-in-Place Concrete Multicell Box	d	<p align="center"><u>One-Cell Box Girder</u></p> <p align="center">Up to One Lane Loaded*</p> $\frac{W}{12}(1.65 - 0.01W)**$ <p align="center">1.3</p> <p align="center">Any Fraction or Number of Lanes:</p> $\frac{W}{12}(1.65 - 0.01W)**$ $\frac{W}{12}(1.5 - 0.014W)$ <p align="center">2.1</p>	$60 \leq L \leq 240$ $35 \leq d \leq 110$ $N_c = 1$  $6 \leq W < 10$  $10 \leq W \leq 24$  $6 \leq W < 12$  $12 \leq W < 20$  $20 \leq W \leq 24$
		<p align="center"><u>Two-Cell Box Girder</u></p> <p align="center">Up to One Lane Loaded*:</p> $1.3 + 0.01(W-12)$ <p align="center">Any Fraction or Number of Lanes:</p> $\frac{W}{12}(1.5 - 0.014W)$	$60 \leq L \leq 240$ $35 \leq d \leq 110$ $N_c = 2$  $12 \leq W \leq 36$  $12 \leq W \leq 36$



# CA Amendment V 4.08.01- Chapter 4

**Table 4.6.2.2.3a-2 Total Design Live Load Lanes for Shear**

Type of Superstructure	Applicable Cross-Section from Table 4.6.2.2.1-1	Total Live Load Design Lanes	Range of Applicability
Cast-in-Place Concrete Multicell Box	d	<u>One-Cell Box Girder</u>  $2 \cdot \left(\frac{S}{4}\right)^{0.4} \left(\frac{d}{12L}\right)^{0.06}$	$60 \leq L \leq 240$ $35 \leq d \leq 110$ $N_c = 1$  $6 \leq S \leq 14$
		<u>Two-Cell Box Girder</u>  $3 \cdot \left(\frac{S}{4.8}\right)^{0.5} \left(\frac{d}{12L}\right)^{0.09}$	$60 \leq L \leq 240$ $35 \leq d \leq 110$ $N_c = 2$  $6 \leq S \leq 14$



# Conclusion

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SAC investigation focuses on live load distribution on one- and two-cell box girders ( $N_c \leq 2$ ):

Girder spacing,  $S$ :  $6' \leq S \leq 13'$

Span length,  $L$ :  $60' \leq L \leq 220'$

Structure depth,  $d$ :  $35'' \leq d \leq 110''$

Aspect ratio,  $d/L$ :  $0.04 \leq d/L \leq 0.06$

Constraints include:

- Constant superstructure cross-section;
- Roadways overhang less than the smaller of  $S/2$  and 5 ft;
- Skew angle is zero degree.
  
- Moment reinforcement is distributed assuming equal stress distribution across the bridge width
- Shear reinforcement is equally distributed to each girder.



## Appendix 1

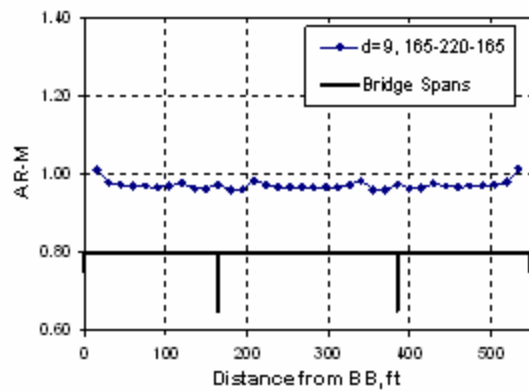
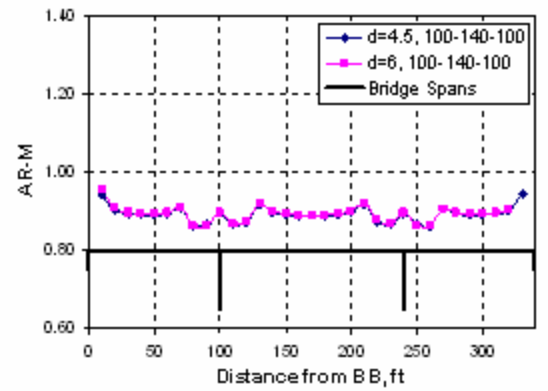
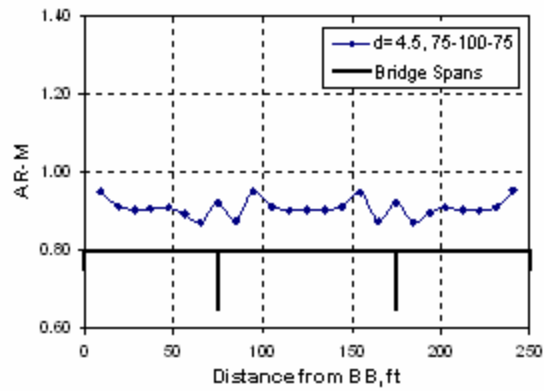
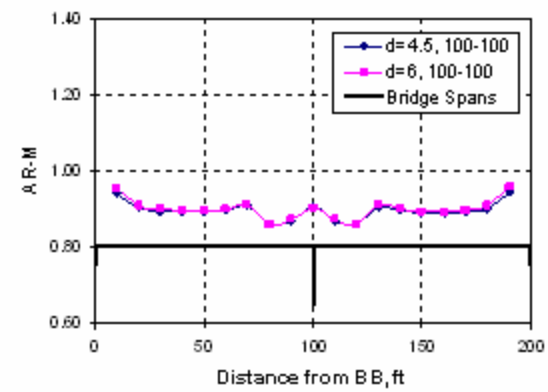
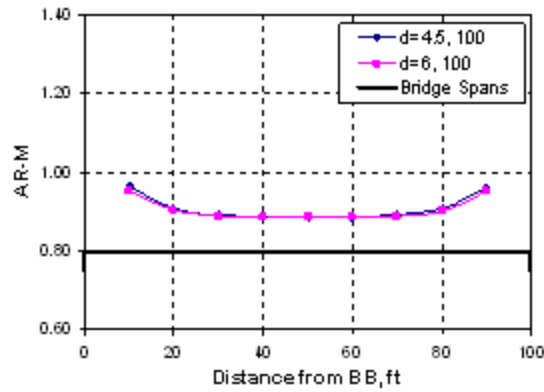
### DISTRIBUTION FACTORS FOR TWO AND THREE SPAN BRIDGES

$$AR = \frac{TLL_{SAP2000}}{TLL_{formula}}, \quad (4-1)$$

where  $TLL_{SAP2000}$  is the TLL computed from SAP2000, and  $TLL_{formula}$  is the TLL computed from proposed equations. Ideally, the AR shall be smaller than but close to one, indicating the formula provides an accurate estimation. An AR larger than one means the TLL computed by the proposed equation is insufficient. If the AR is less than 0.8, it is considered that the proposed equation is overly conservative.

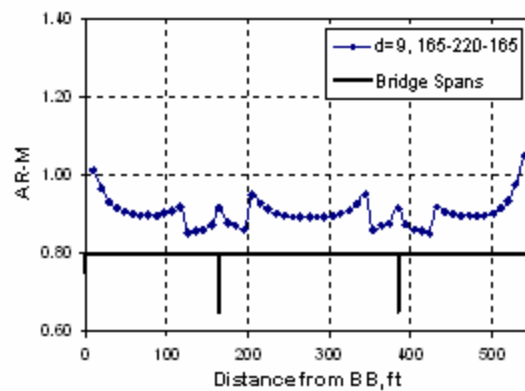
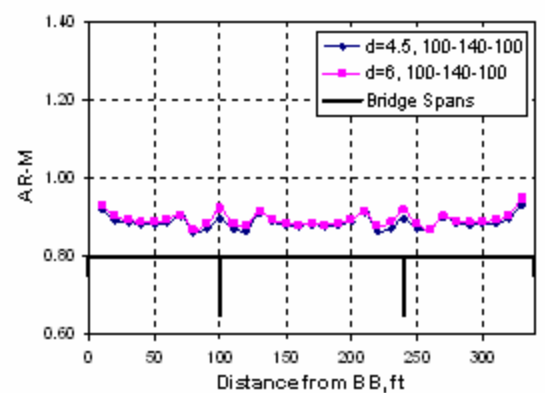
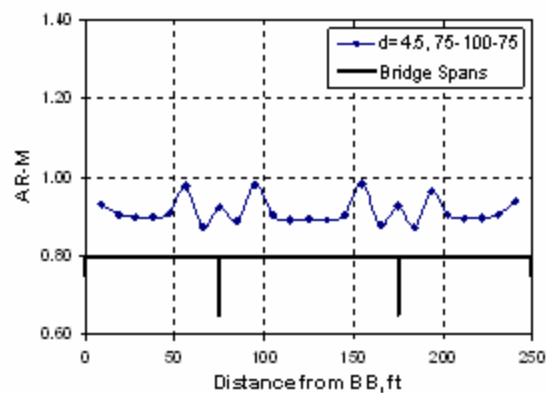
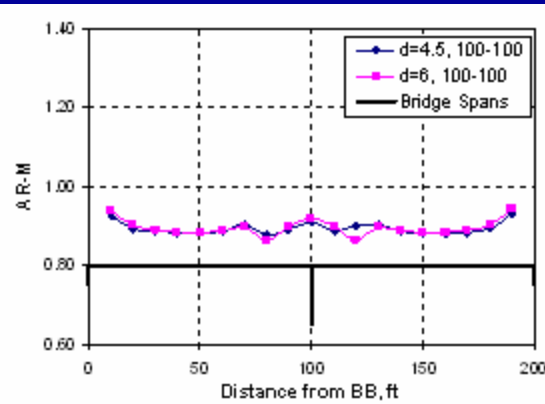
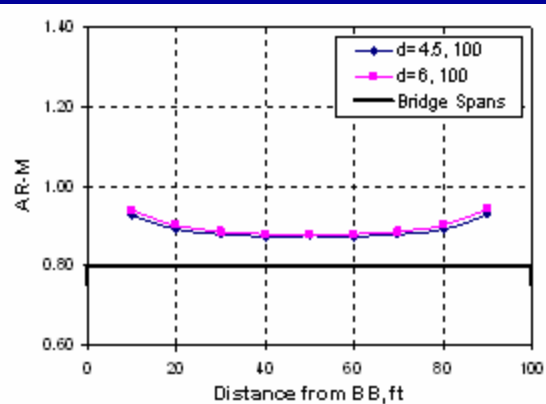


# Appendix 1





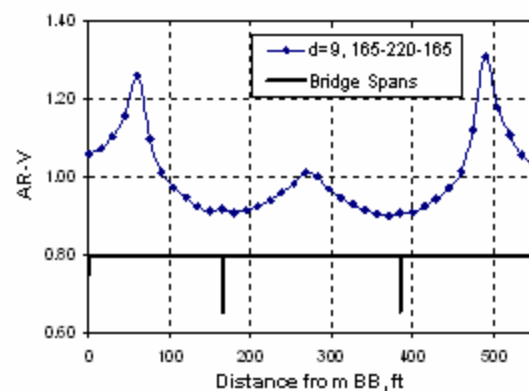
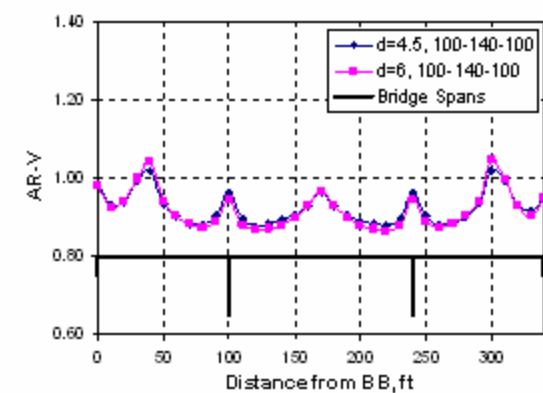
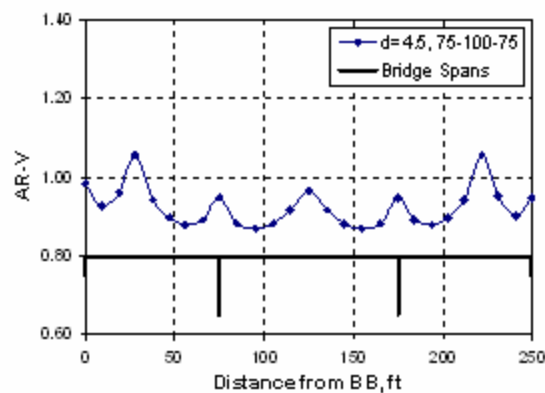
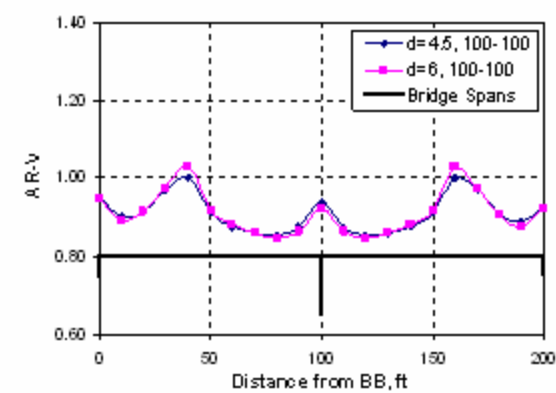
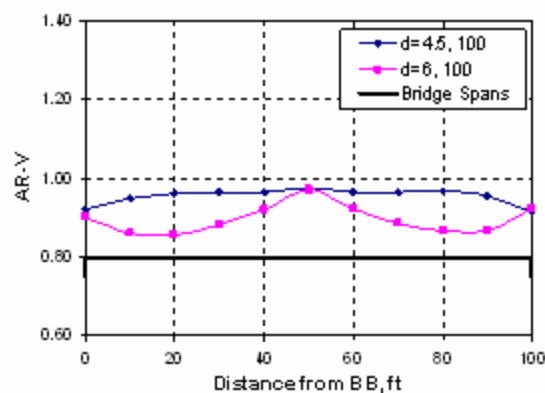
# Appendix 1





## Appendix 1

Figure 4.4: Acceptance ratio of shear TLL for bridges of width = 24 ft; legend: “d=4.5, 100-100” means the superstructure depth, d = 4.5 ft and the span length, L, are 100 ft, 100 ft

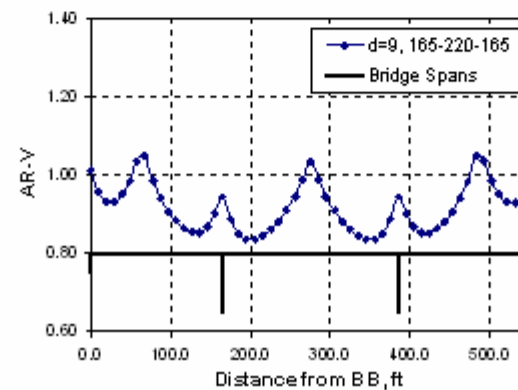
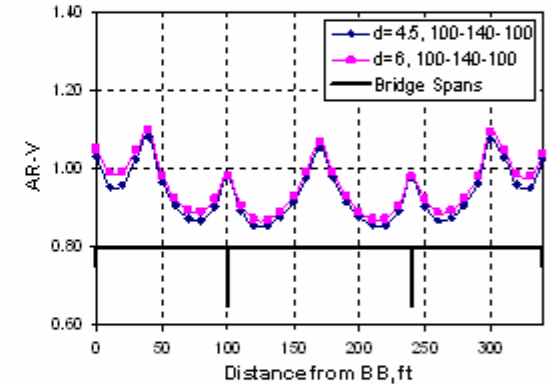
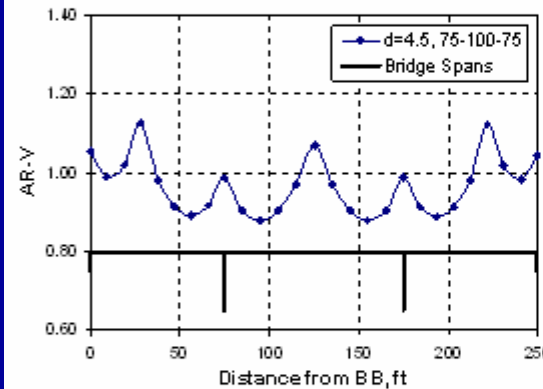
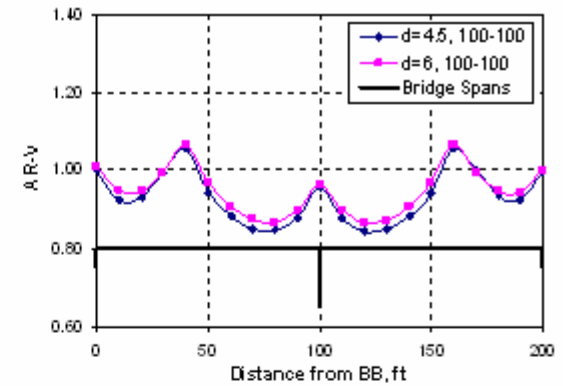
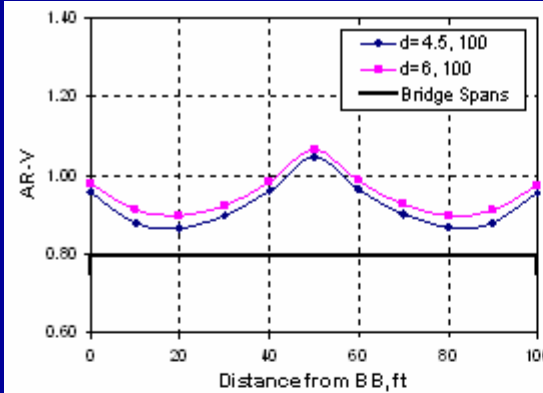






# Appendix 1

Figure 4.5: Acceptance ratio of shear TLL for bridges of width = 36 ft; legend: “d=4.5, 100-100” means the superstructure depth, d = 4.5 ft and the span length, L, are 100 ft, 100 ft





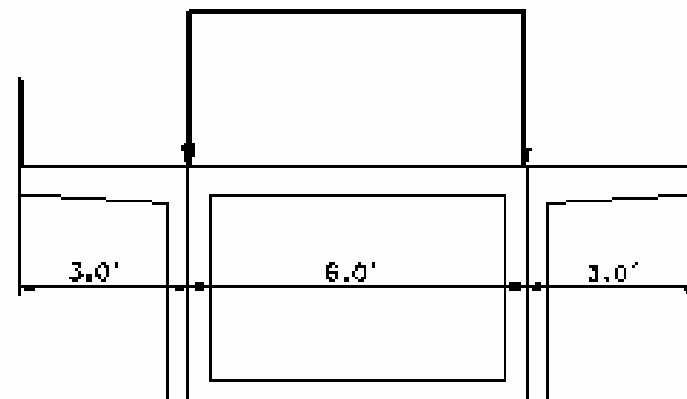
## Appendix 2- Case Study

### APPENDIX A – SAP2000 RESULTS

#### A.1 BRIDGE GROUP 1: ONE CELL BOX GIRDER BRIDGE

A.1.1 Girder Spacing = 6', Maximum Number of Lanes = 1

#	Cell Type	L (ft)	D (ft)
1	1A3	65	3.9
2	1A4	100	4
3	1A5	100	5
5	1A6	100	6
8	1A6	150	6



X1A

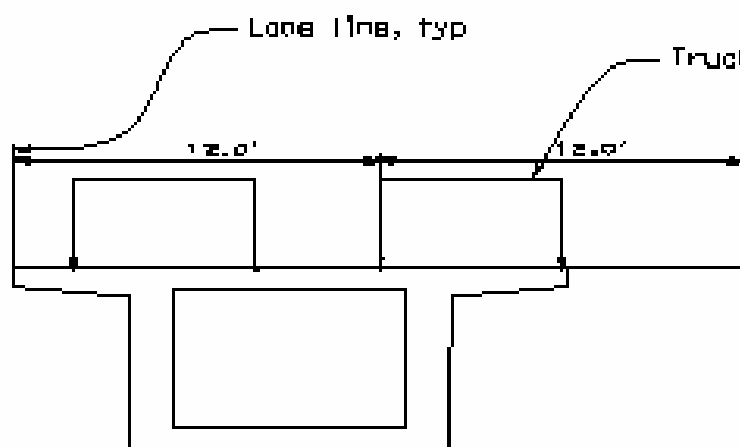
td = 7.125'  
te = 6.0'  
tg = 12.0'



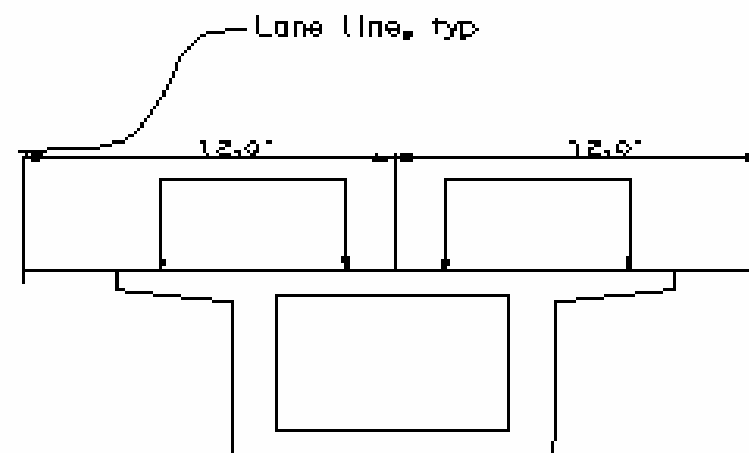
# Engineering Services- Structural Analysis Committee

b)  $1 < \# \text{ Lanes} < 2$

#	Cell Type	L (ft)	D (ft)
1	1B5	100	5



X1B  $t_d = 8.125''$   
 $t_g = 7.5''$   
 $t_g = 12.0''$



X1B  $t_d = 8.125''$   
 $t_g = 7.5''$   
 $t_g = 12.0''$

A.1.3 Girder Spacing = 8', Maximum Number of Lanes < 2

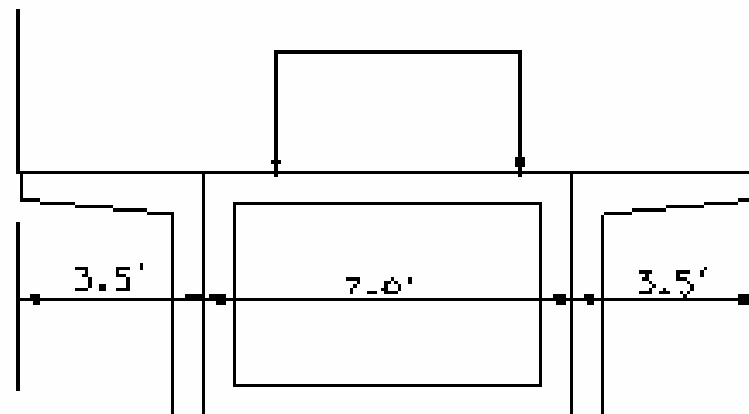


## Engineering Services- Structural Analysis Committee

A.1.2 Girder Spacing = 7', Maximum Number of Lanes < 2

#	Cell Type	L (ft)	D (ft)
1	1B5	100	5

a) 1 Lane



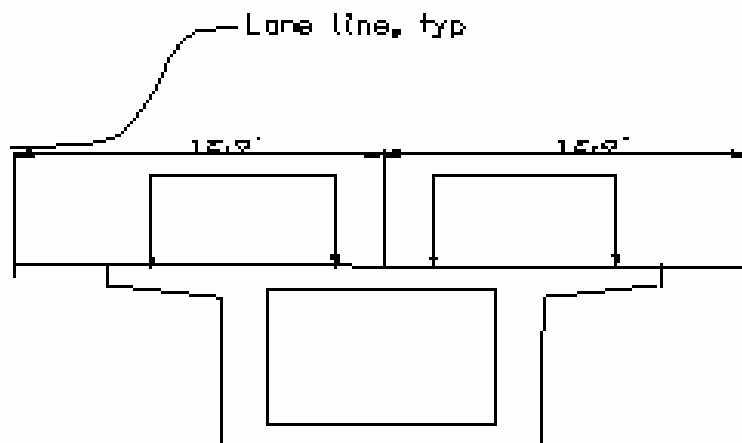
X1B     $t_d = 8.125'$   
           $t_s = 7.5''$   
           $t_g = 12.0''$



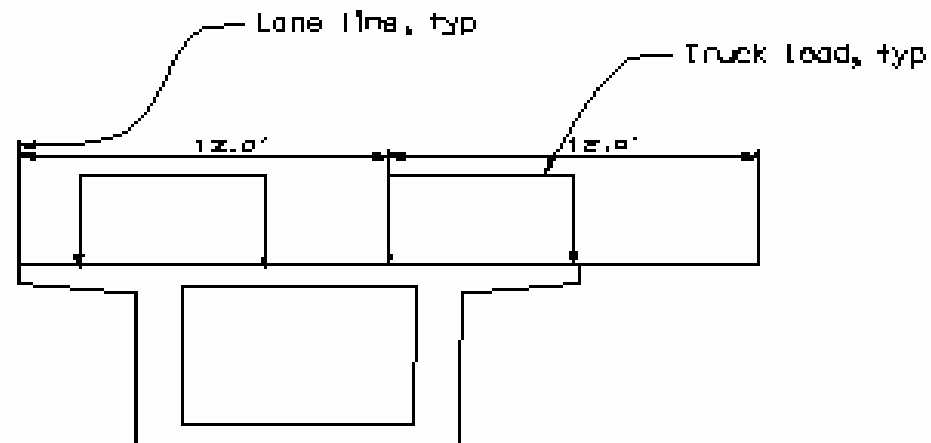
# Engineering Services- Structural Analysis Committee

b)  $1 < \# \text{ Lanes} < 2$

#	Cell Type	L (ft)	D (ft)
1	1B5	100	5



X1B  $t_d = 8.125'$   
 $t_g = 7.5'$   
 $t_g = 12.0'$



X1B  $t_d = 8.125'$   
 $t_g = 7.5'$   
 $t_g = 12.0'$

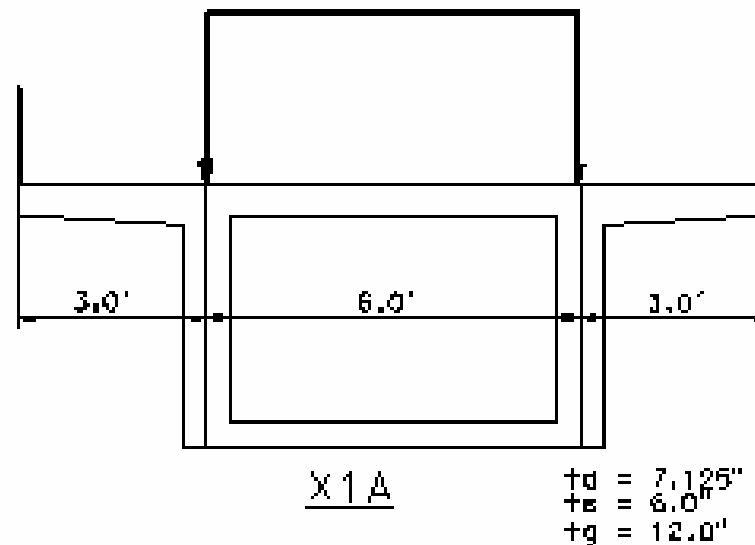


## Engineering Services- Structural Analysis Committee

A.1.5 Girder Spacing = 9', Maximum Number of Lanes = 2

#	Cell Type	L (ft)	D (ft)
4-1	1B5	100	5
6-1	1B6	100	6
9-1	1B6	150	6
11-1	1B8	180	8
13-1	1B9	220	9.17

a) 1 Lane

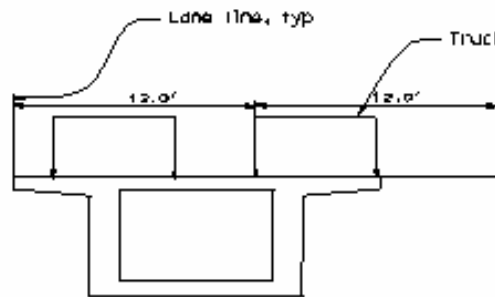




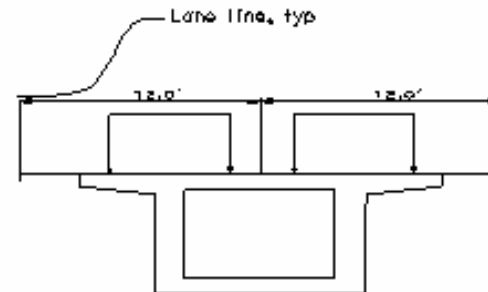
# Engineering Services- Structural Analysis Committee

b)  $1 < \# \text{ Lanes} < 2$

#	Cell Type	L (ft)	D (ft)
1	1B5	100	5

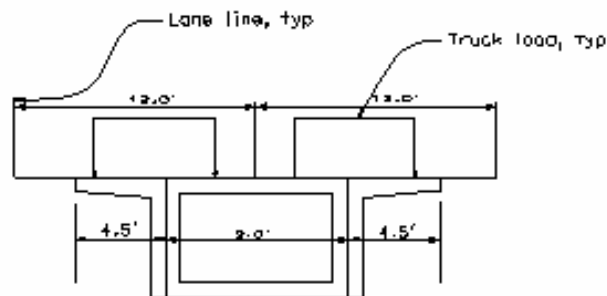


X1B  $t_d = 8.125'$   
 $t_s = 7.5'$   
 $t_g = 12.0'$

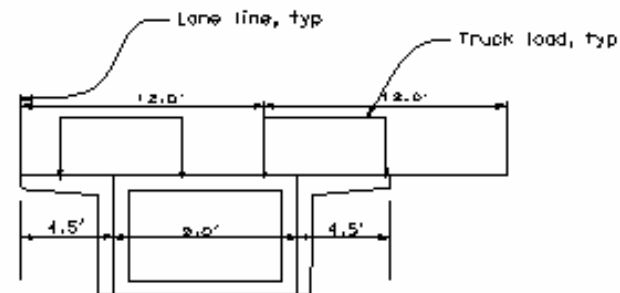


X1B  $t_d = 8.125'$   
 $t_s = 7.5'$   
 $t_g = 12.0'$

c) 2 Lanes



X1B  $t_d = 8.125'$   
 $t_s = 7.5'$   
 $t_g = 12.0'$



X1B  $t_d = 8.125'$   
 $t_s = 7.5'$   
 $t_g = 12.0'$



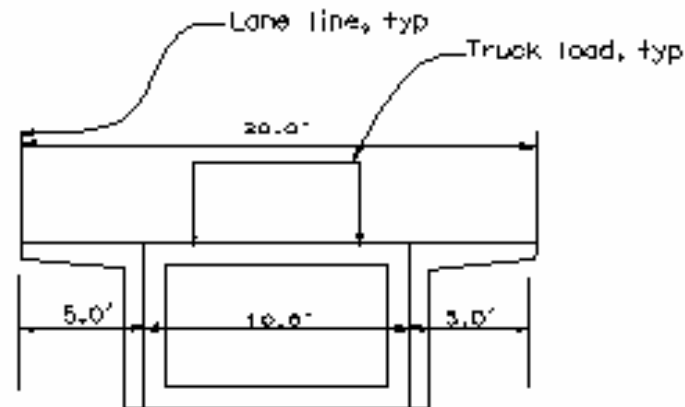
# Engineering Services- Structural Analysis Committee

A.1.7

Girder Spacing = 10', Maximum Number of Lanes= 2

#	Cell Type	L (ft)	D (ft)
4	1B5	100	5
6	1B6	100	6
9	1B6	150	6
11	1B8	180	8
13	1B9	220	9.17

a) 1 Lane



X1B  $t_d = 8.125''$   
 $t_s = 7.5''$   
 $t_g = 12.0''$



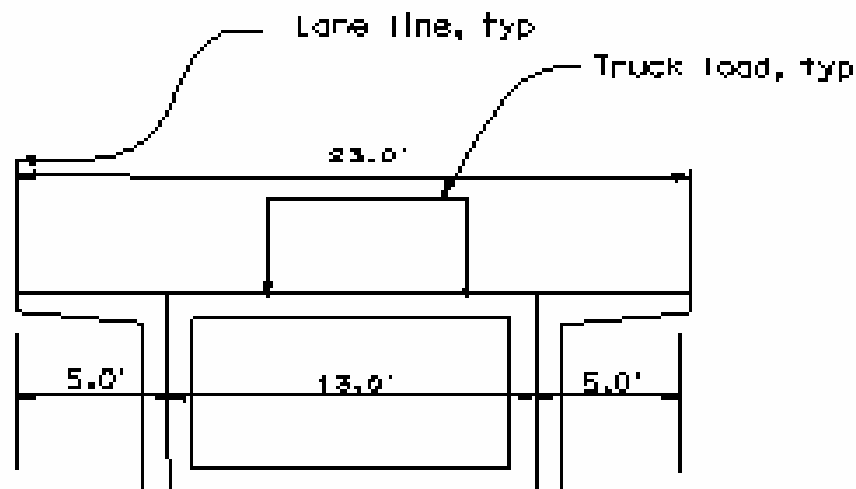


## Engineering Services- Structural Analysis Committee

A.1.8 Girder Spacing = 13', Maximum Number of Lanes = 2

#	Cell Type	L (ft)	D (ft)
7	1C6	100	5
10	1C6	100	6
12	1C8	150	6
14	1C9	180	8

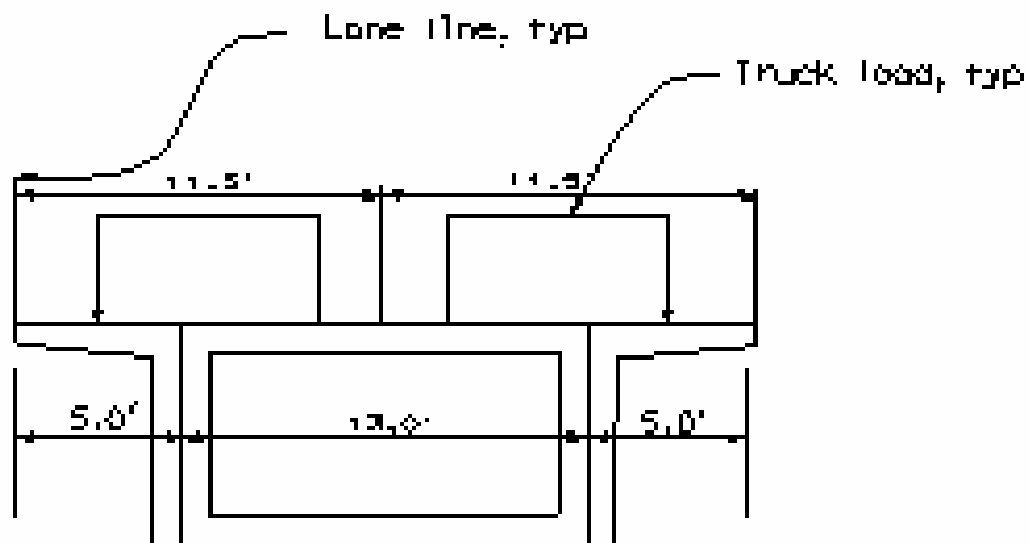
a) 1 Lane



X1C  $t_d = 9.5'$   
 $t_s = 9.75'$   
 $t_g = 12.0'$



b) 2 Lanes



X1C  $t_d = 9.5''$   
 $t_s = 9.75''$   
 $t_g = 12.0''$

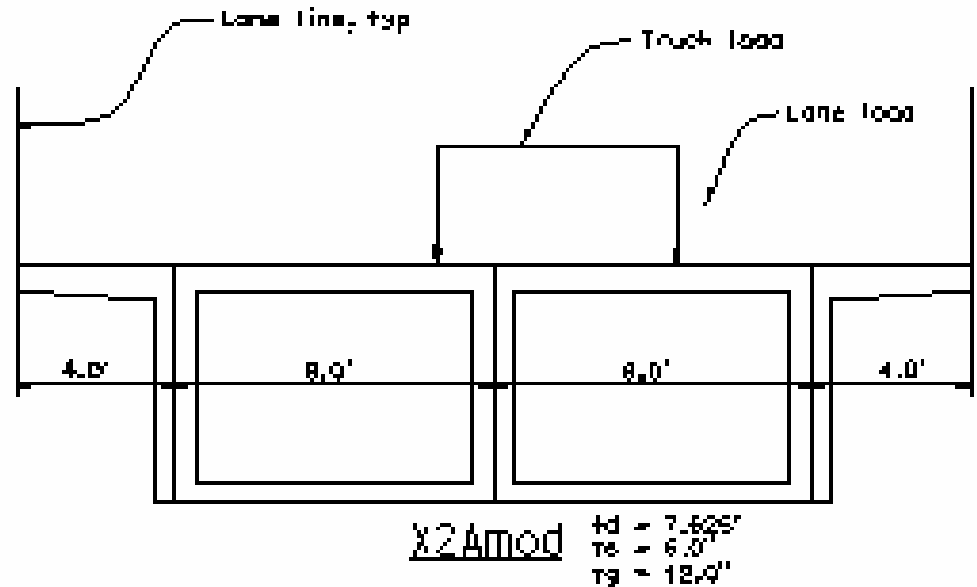


## Engineering Services- Structural Analysis Committee

A.2.2 Girder Spacing = 8', Maximum Number of Lanes = 2

#	Cell Type	L (ft)	D (ft)
2	2A4	75	4

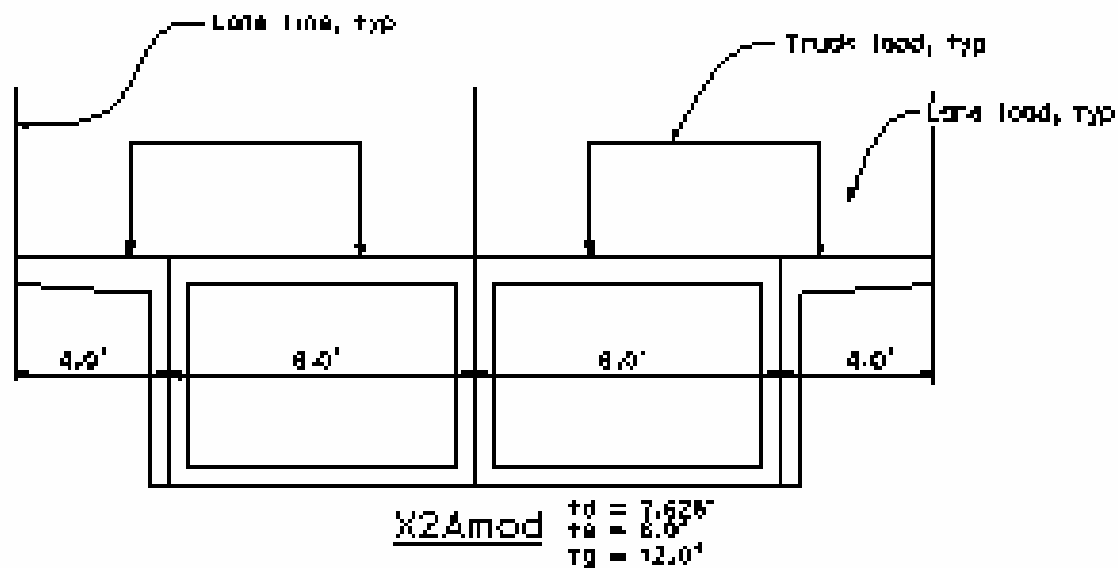
a) 1 Lane





# Engineering Services- Structural Analysis Committee

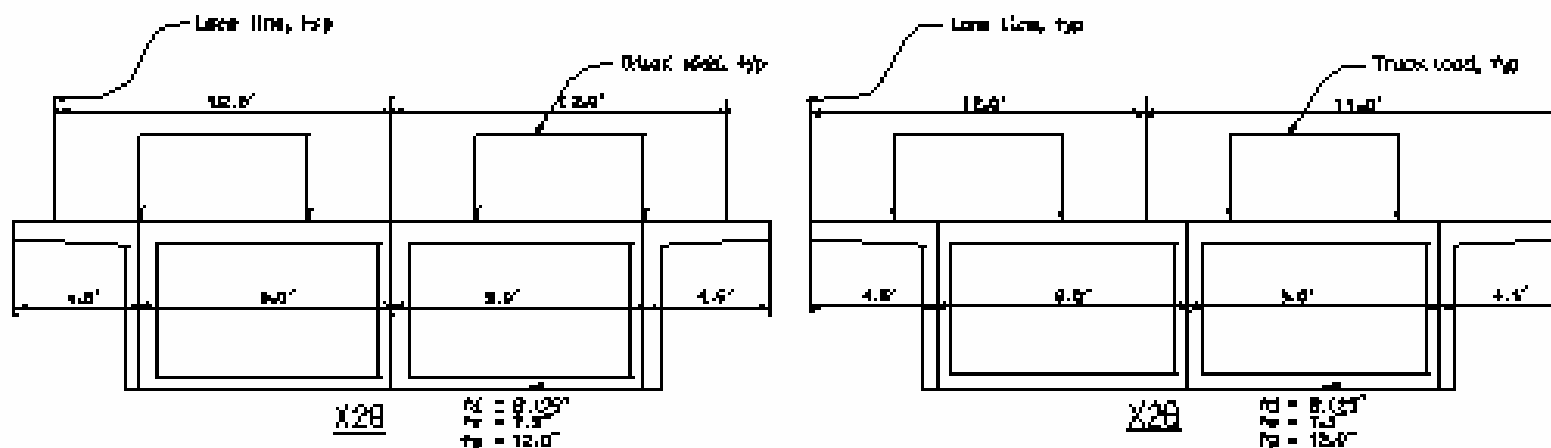
b) 2 Lanes





# Engineering Services- Structural Analysis Committee

a) 2 Lanes





## Engineering Services- Structural Analysis Committee

A.2.4 Girder Spacing = 10', Maximum Number of Lanes= 2

#	Cell Type	L (ft)	D (ft)
4	2B5	100	5
6	2B6	100	6
9	2B6	150	6
11	2B8	180	8
13	2B9	220	9.17

a) 1 Lane

