AASHTO LRFD Shear: A Plethora of Choices

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Western Bridge Engineer's Seminar

9/24/07

Outline

- Timeline of changes by AASHTO SCOBS T10 Tech. Comm.-Concrete
- Describe each method; give pros, cons
- (will skip shear friction §5.8.4)
- Longitudinal steel (sectional methods)
- Torsion
- Recommendations w/disclaimer

Timeline

- ‘9X “Summit Meeting of Shear Experts”
- ’94 1st Ed.—Strut-and-Tie plus:
  - Sectional method--MCFT using β θ Tables
  - For non-PS sections, $\beta = 2.0$ and $\theta = 45^o$
- ’98 2nd Ed., ’99 Interims--Strain revised, measured at mid-height – changes by 50%
- ’04 3rd Ed., ’05 Interims—Principal f_t limit of $0.11\sqrt{f'_c}$, methodology from Seg. Guide Specs
- ’07 4th Ed.—modified V_{ci}, V_{cw} (NCHRP 549)
- ’07 4th Ed., ’08 Interims--Canadian shear (“)
- ’07--New Seismic Guide Specs w/ductility method

The Choices

- Sectional methods
  - Simplified MCFT w/tabularized θ, β
  - Simplified MCFT w/ $\theta = 45$; $\beta = 2.0$ (§5.8.3.4.1)
  - Simplified MCFT w/eqns for θ, β (§5.8.3.4.2)
  - Option to use modified V_{ci} , V_{cw} (§5.8.3.4.3)
  - Option for segmental bridges (§5.8.6)
- Member methods
  - *AASHTO LRFD Seismic Guide Specs*
  - Strut-and-Tie (§5.6.3)

Tabularized “θ, β” Method

- Simplified form of modified compression field theory (MCFT) by Collins, Mitchell—*a complete behavior model*
- V_c is dependent on the applied ε and v_u by adjusting β in $V_c=0.0316\beta\sqrt{f'_c}b_vd_v$
- Angle of diagonal compressive stress considered in stirrups: $$V_s = \frac{A_v f_y d_v(\cot\theta + \cot\alpha)\sin\alpha}{s}$$

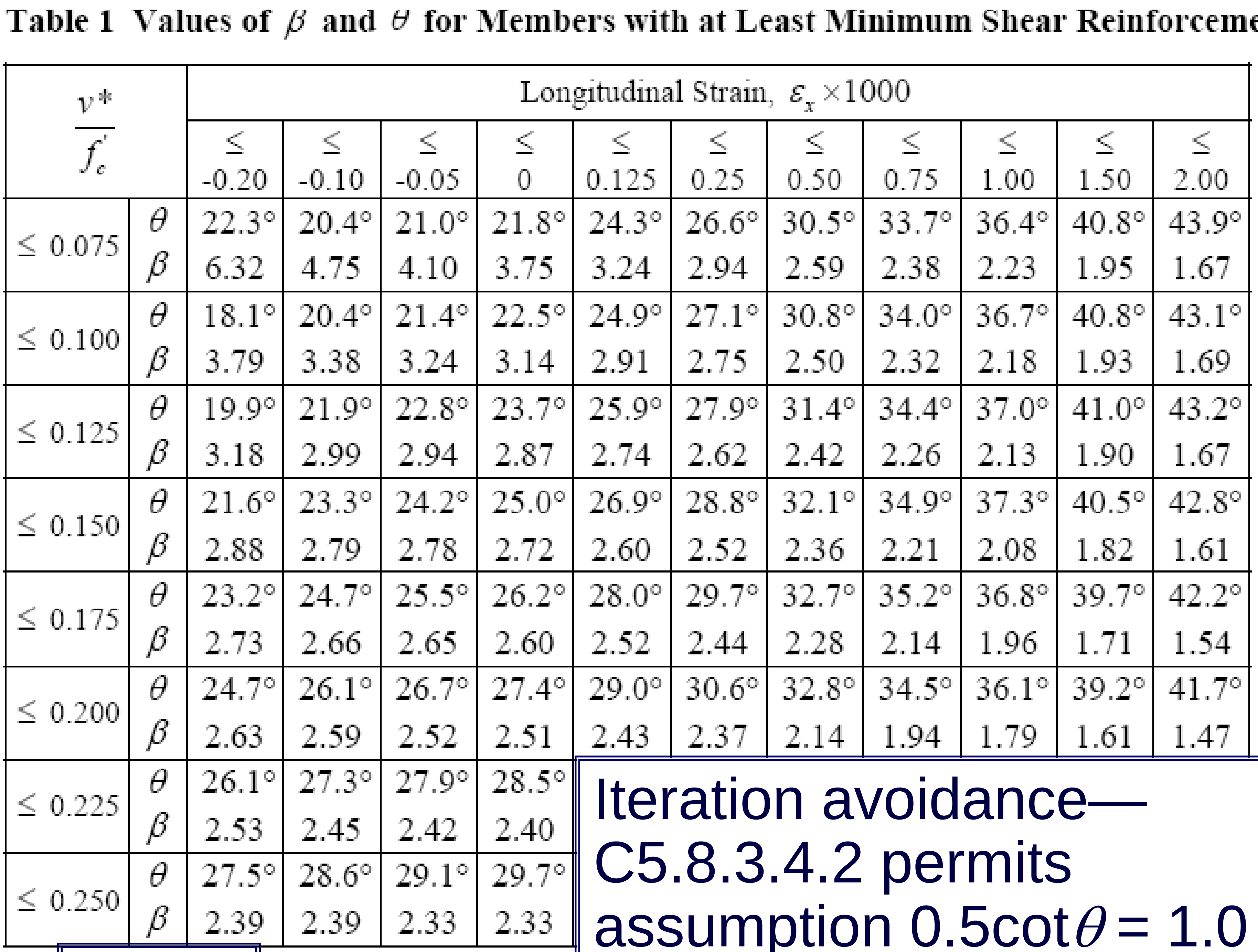
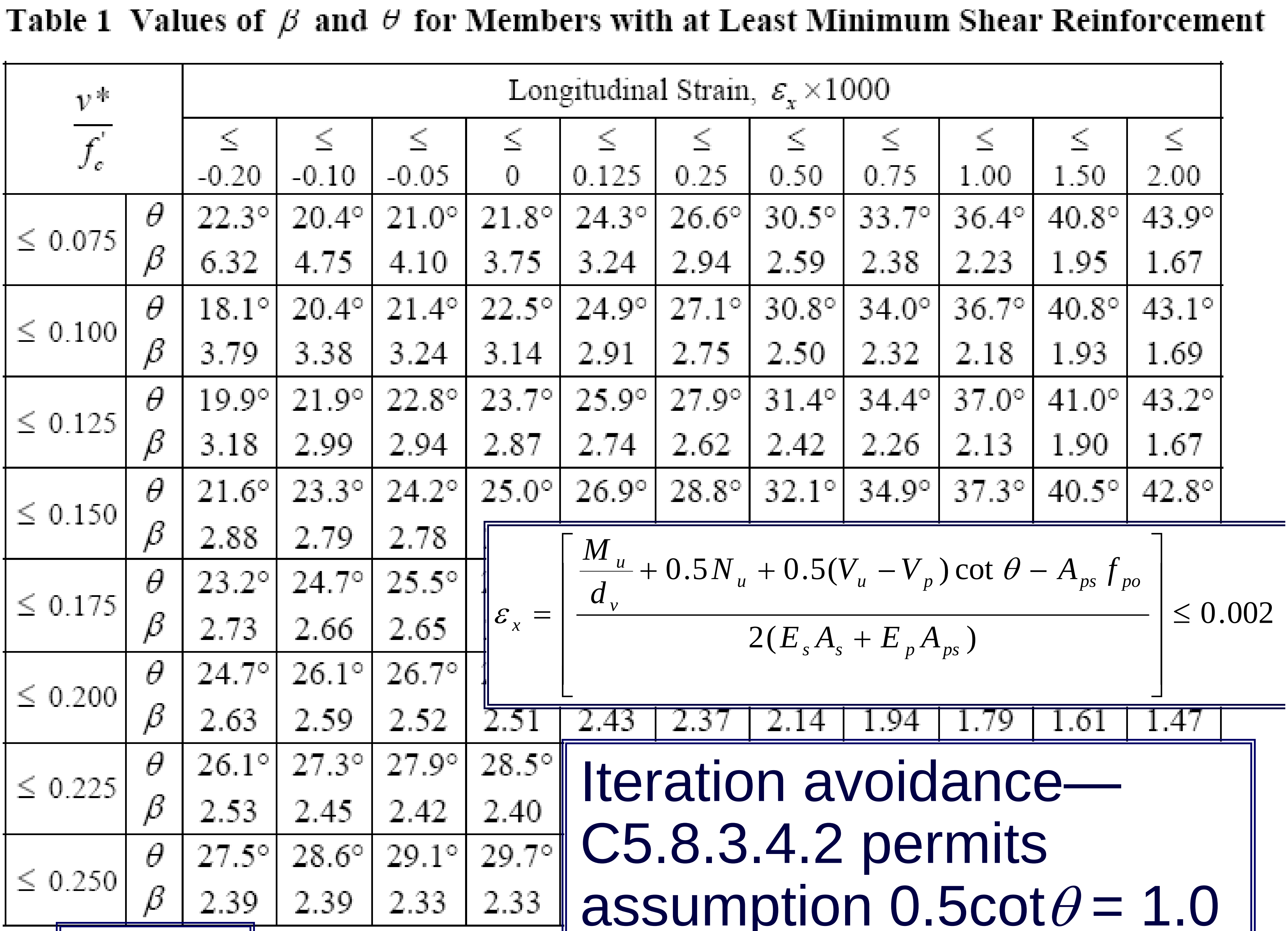


Table 1 Values of β and θ for Members with at Least Minimum Shear Reinforcement

$\frac{v^*}{f'_c}$		Longitudinal Strain, $\varepsilon_x \times 1000$										
		≤ -0.20	≤ -0.10	≤ -0.05	≤ 0	≤ 0.125	≤ 0.25	≤ 0.50	≤ 0.75	≤ 1.00	≤ 1.50	≤ 2.00
≤ 0.075	θ	22.3°	20.4°	21.0°	21.8°	24.3°	26.6°	30.5°	33.7°	36.4°	40.8°	43.9°
	β	6.32	4.75	4.10	3.75	3.24	2.94	2.59	2.38	2.23	1.95	1.67
≤ 0.100	θ	18.1°	20.4°	21.4°	22.5°	24.9°	27.1°	30.8°	34.0°	36.7°	40.8°	43.1°
	β	3.79	3.38	3.24	3.14	2.91	2.75	2.50	2.32	2.18	1.93	1.69
≤ 0.125	θ	19.9°	21.9°	22.8°	23.7°	25.9°	27.9°	31.4°	34.4°	37.0°	41.0°	43.2°
	β	3.18	2.99	2.94	2.87	2.74	2.62	2.42	2.26	2.13	1.90	1.67
≤ 0.150	θ	21.6°	23.3°	24.2°	25.0°	26.9°	28.8°	32.1°	34.9°	37.3°	40.5°	42.8°
	β	2.88	2.79	2.78	2.72	2.60	2.52	2.36	2.21	2.08	1.82	1.61
≤ 0.175	θ	23.2°	24.7°	25.5°	26.2°	28.0°	29.7°	32.7°	35.2°	36.8°	39.7°	42.2°
	β	2.73	2.66	2.65	2.60	2.52	2.44	2.28	2.14	1.96	1.71	1.54
≤ 0.200	θ	24.7°	26.1°	26.7°	27.4°	29.0°	30.6°	32.8°	34.5°	36.1°	39.2°	41.7°
	β	2.63	2.59	2.52	2.51	2.43	2.37	2.14	1.94	1.79	1.61	1.47
≤ 0.225	θ	26.1°	27.3°	27.9°	28.5°							
	β	2.53	2.45	2.42	2.40							
≤ 0.250	θ	27.5°	28.6°	29.1°	29.7°							
	β	2.39	2.39	2.33	2.33							

$^*v = V_u/b_v d_v$

Iteration avoidance—C5.8.3.4.2 permits assumption $0.5\cot\theta = 1.0$



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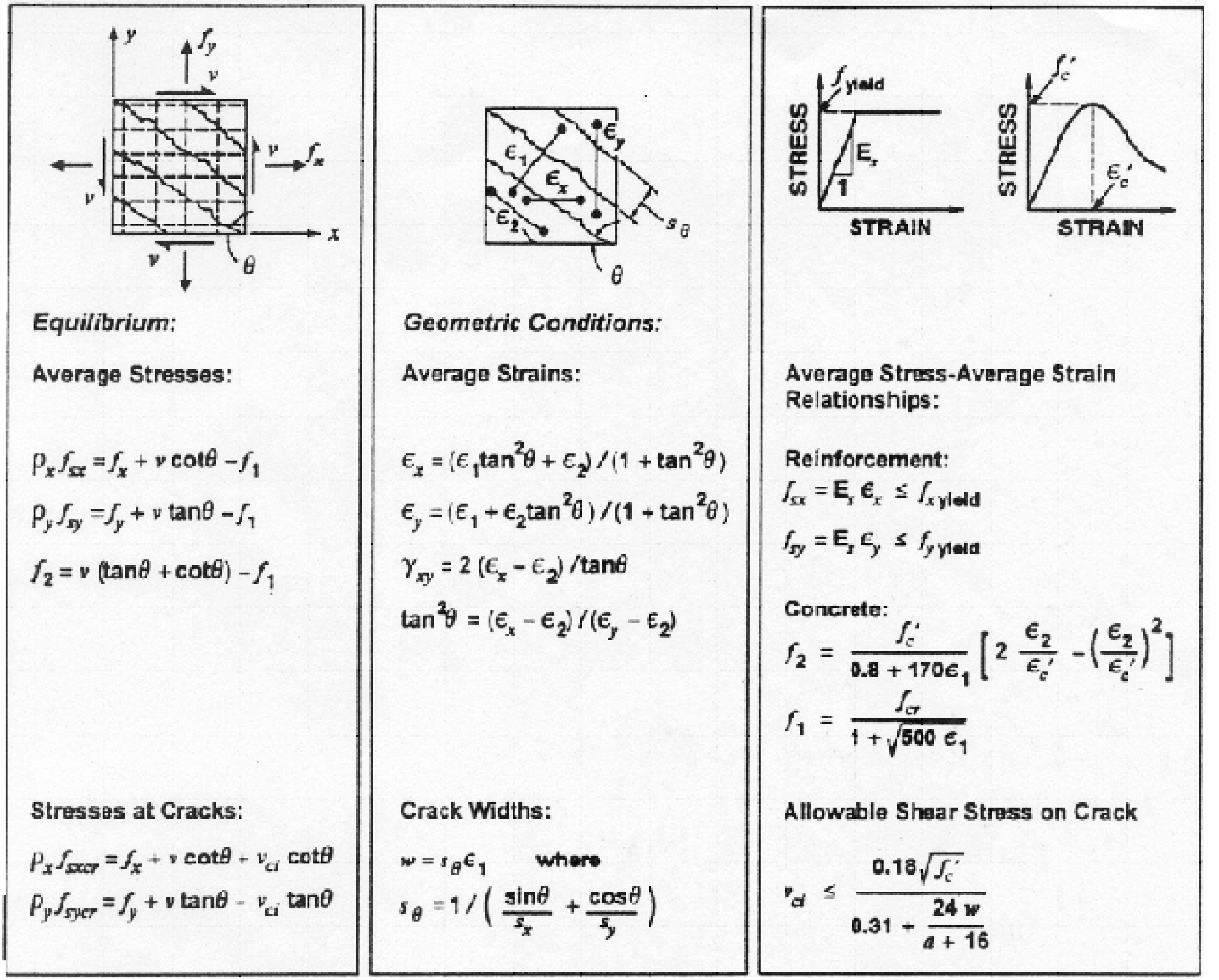
$^*v = V_u/b_v d_v$

$$\varepsilon_x = \left[\frac{\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps} f_{po}}{2(E_s A_s + E_p A_{ps})} \right] \leq 0.002$$

Iteration avoidance—C5.8.3.4.2 permits assumption $0.5\cot\theta = 1.0$



Equilibrium:

Average Stresses:

$$\rho_x f_{sx} = f_x + v\cot\theta - f_1$$

$$\rho_y f_{sy} = f_y + v\tan\theta - f_1$$

$$f_2 = v(\tan\theta + \cot\theta) - f_1$$

Stresses at Cracks:

$$\rho_x f_{sxcr} = f_x + v\cot\theta + v_{ci}\cot\theta$$

$$\rho_y f_{sycr} = f_y + v\tan\theta - v_{ci}\tan\theta$$

Geometric Conditions:

Average Strains:

$$\epsilon_x = (\epsilon_1\tan^2\theta + \epsilon_2)/(1 + \tan^2\theta)$$

$$\epsilon_y = (\epsilon_1 + \epsilon_2\tan^2\theta)/(1 + \tan^2\theta)$$

$$\gamma_{xy} = 2(\epsilon_x - \epsilon_2)/\tan\theta$$

$$\tan^2\theta = (\epsilon_x - \epsilon_2)/(\epsilon_y - \epsilon_2)$$

Crack Widths:

$w = s_\theta \epsilon_1$ **where**

$$s_\theta = 1 \Big/ \left(\frac{\sin\theta}{s_x} + \frac{\cos\theta}{s_y}\right)$$

Average Stress-Average Strain Relationships:

Reinforcement:

$$f_{sx} = E_s\epsilon_x \le f_{x\,yield}$$

$$f_{sy} = E_s\epsilon_y \le f_{y\,yield}$$

Concrete:

$$f_2 = \frac{f_c'}{0.8 + 170\epsilon_1}\left[2\frac{\epsilon_2}{\epsilon_c'} - \left(\frac{\epsilon_2}{\epsilon_c'}\right)^2\right]$$

$$f_1 = \frac{f_{cr}}{1 + \sqrt{500\,\epsilon_1}}$$

Allowable Shear Stress on Crack

$$v_{ci} \le \frac{0.18\sqrt{f_c'}}{0.31 + \dfrac{24w}{a + 16}}$$

Figure 12 Summary of Relationships Used in the Modified Compression Field Theory

$\theta = 45, \beta = 2.0, \alpha = 90^{o}$

- Non-PS members
- Components less than 16-in. thick
- $V_c = 0.0316 \beta \sqrt{f'_c} b_v d_v$
- $V_s = V_u - \phi V_c$

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s}$$

- Note—some DOT’s using for all members!

Explicit Eqns for “θ, β”

- Direct solution—Canadian Stds Assoc., 2004, $V_c = \beta\sqrt{f'_c}\,b_v d_v; \quad \beta = \dfrac{4.8}{1+1500\varepsilon_x}$

$$V_s = \frac{A_v f_y d_v \cot\theta}{s} \quad \textit{assume} \quad \textit{vertical} \quad \textit{stirrups}; \quad \theta = 29 + 7000\varepsilon_x$$

- AASHTO ’08—adopted “Canadian” equations, moved β–θ Tables to App. (Caltrans--continue using β–θ Tables)

Modified Std. Specs--or “V_{ci}, V_{cw} Method”

- 4th Ed. (’07)— V_{ci} and V_{cw} (§5.8.3.4.3)

$$V_{ci} = 0.632\sqrt{f'_c}\,b_v d_v + V_d + \frac{V_i M_{cr}}{M_{\max}} \geq 1.9\sqrt{f'_c}\,b_v d$$

$$V_{cw} = \left(1.9\sqrt{f'_c} + 0.30 f_{pc}\right) b_v d_v + V_p$$

Kuchma and Hawkins, “Simplified Shear Design of Concrete Members” NCHRP Report No. 549, 2005

Pros

- Familiarity w/past
- Ancestry never had iteration
- Incipient cracking more appropriate for Service

Cons

- Not a complete mechanistic model
- Less accurate per calcs along many components
- Not the tool of choice when “sharpening the pencil”

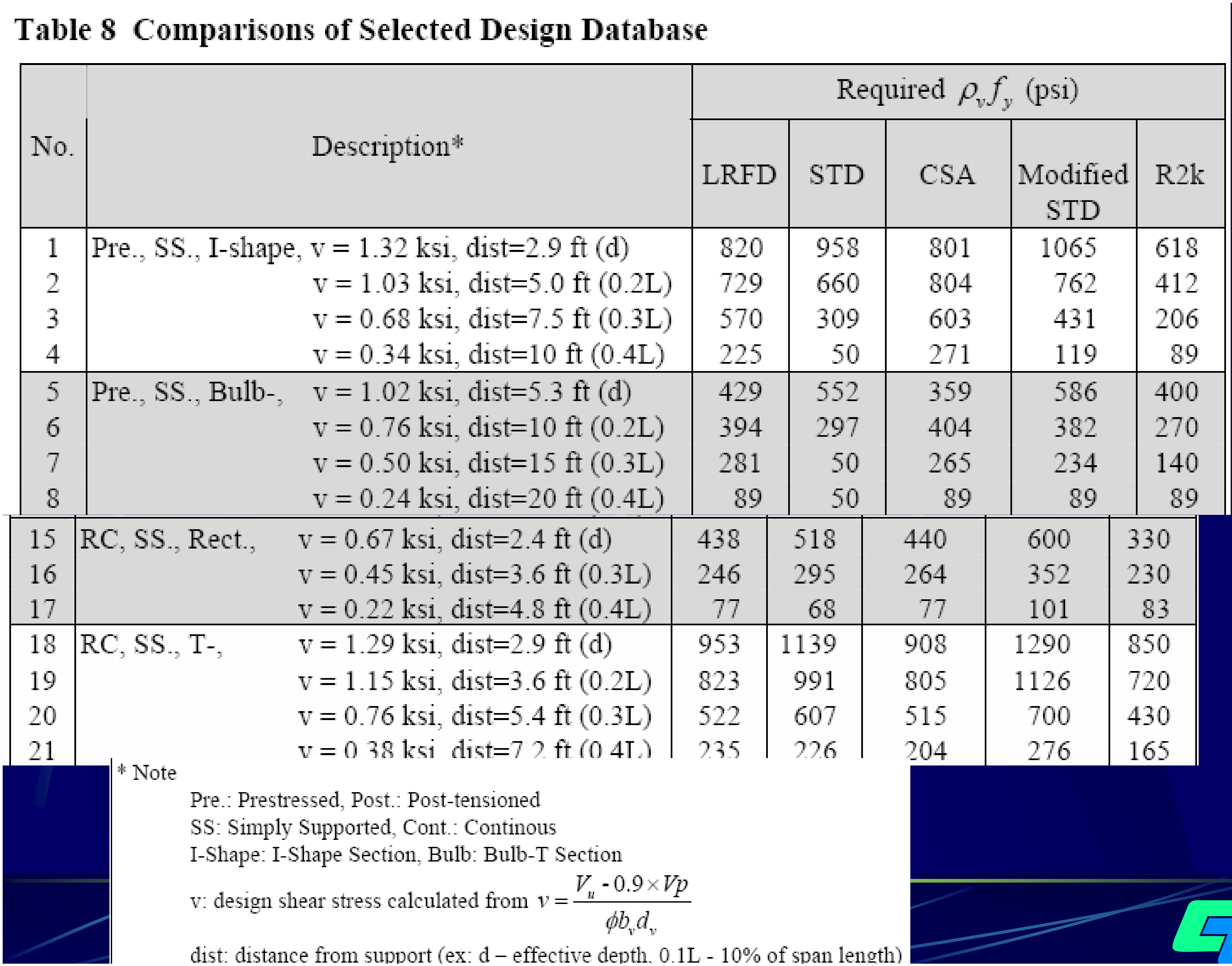


Table 8 Comparisons of Selected Design Database

No.	Description*	Required $\rho_v f_y$ (psi)				
		LRFD	STD	CSA	Modified STD	R2k
1	Pre., SS., I-shape, v = 1.32 ksi, dist=2.9 ft (d)	820	958	801	1065	618
2	v = 1.03 ksi, dist=5.0 ft (0.2L)	729	660	804	762	412
3	v = 0.68 ksi, dist=7.5 ft (0.3L)	570	309	603	431	206
4	v = 0.34 ksi, dist=10 ft (0.4L)	225	50	271	119	89
5	Pre., SS., Bulb-, v = 1.02 ksi, dist=5.3 ft (d)	429	552	359	586	400
6	v = 0.76 ksi, dist=10 ft (0.2L)	394	297	404	382	270
7	v = 0.50 ksi, dist=15 ft (0.3L)	281	50	265	234	140
8	v = 0.24 ksi, dist=20 ft (0.4L)	89	50	89	89	89
15	RC, SS., Rect., v = 0.67 ksi, dist=2.4 ft (d)	438	518	440	600	330
16	v = 0.45 ksi, dist=3.6 ft (0.3L)	246	295	264	352	230
17	v = 0.22 ksi, dist=4.8 ft (0.4L)	77	68	77	101	83
18	RC, SS., T-, v = 1.29 ksi, dist=2.9 ft (d)	953	1139	908	1290	850
19	v = 1.15 ksi, dist=3.6 ft (0.2L)	823	991	805	1126	720
20	v = 0.76 ksi, dist=5.4 ft (0.3L)	522	607	515	700	430
21	v = 0.38 ksi, dist=7.2 ft (0.4L)	235	226	204	276	165

* Note

Pre.: Prestressed, Post.: Post-tensioned
SS: Simply Supported, Cont.: Continous
I-Shape: I-Shape Section, Bulb: Bulb-T Section

v: design shear stress calculated from $v = \dfrac{V_u - 0.9 \times Vp}{\phi b_v d_v}$

dist: distance from support (ex: d – effective depth, 0.1L - 10% of span length)

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No.	Description*	Required $\rho_v f_y$ (psi)				
		LRFD	STD	CSA	Modified STD	R2k
9	Post., Cont., Box, v = 0.66 ksi, dist=5.2 ft (d)	115	50	77	167	125
10	v = 0.35 ksi, dist=28 ft (0.2L)	77	100	77	111	99
11	v = 0.02 ksi, dist=56 ft (0.4L)	77	50	77	77	77
12	v = 0.40 ksi, dist=84 ft (0.6L)	77	113	77	125	104
13	v = 0.78 ksi, dist=112ft (0.8L)	164	50	77	206	150
14	v = 1.07 ksi, dist=135 ft (L-d)	637	473	630	808	583
22	RC, Cont., Rect., v = 0.29 ksi, dist=2.7 ft (d)	112	138	131	178	158
23	v = 0.23 ksi, dist=3.6 ft (0.2L)	77	71	80	104	138
24	v = 0.03 ksi, dist=7.2 ft (0.4L)	77	50	77	77	77
25	v = 0.29ksi, dist=10.8ft (0.6L)	110	135	129	176	158
26	v = 0.55ksi, dist=14.4ft (0.8L)	297	393	309	462	333
27	v = 0.62 ksi, dist=15.3ft (L-d)	391	460	377	537	396

* Note

Pre.: Prestressed, Post.: Post-tensioned

SS: Simply Supported, Cont.: Continous

I-Shape: I-Shape Section, Bulb: Bulb-T Section

v: design shear stress calculated from $v = \dfrac{V_u - 0.9 \times Vp}{\phi b_v d_v}$

dist: distance from support (ex: d – effective depth, 0.1L - 10% of span length)

Segmental Shear §5.8.6

- Taken from '89 and '99 Guide Specs for Dgn of Segmental Bridges—similar to Std. Specs
- Lessor of: $V_n = V_c + V_s; \quad V_n = 0.379\sqrt{f'_c}\,b_v d_v$

where $V_c = 0.632K\sqrt{f'_c}\,b_v d_v$ and $K = \sqrt{1 + \frac{f_{pc}}{0.0632\sqrt{f'_c}}} \leq 2.0$

f_{pc} = unfactored compressive stress in concrete after prestress losses have occurred either at the centroid of the cross-section resisting transient loads or at the junction of the web and flange where the centroid lies in the flange

Longitudinal Steel Check

$$A_{ps} f_{pu} + A_s f_y \ge \left|\frac{M_u}{d_v \phi}\right| + 0.5\frac{N_u}{\phi} + \left(\left|\frac{V_u}{\phi_v} - V_p\right| - 0.5V_s\right)\cot\theta$$

- Open issues w/Eq 5.8.3.5-1
  - Use max M_u and max V_u ? Or, max M_u and assoc. V_u? Max V_u and assoc. M_u?
  - Which girder distribution factors (gdf)?
  - Apply skew factors? M-V interaction not considered in their development; exacerbated w/skewed support conditions
- Caltrans….

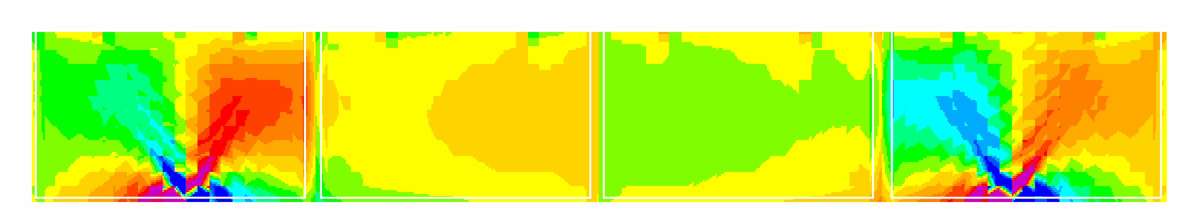
Strut-and-Tie (§5.6.3, §5.8.3)

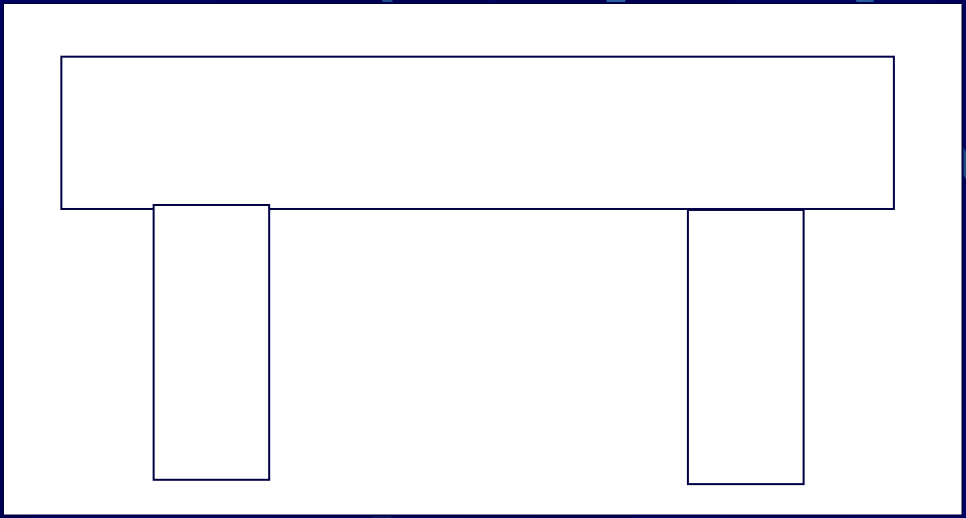
- Simultaneous flexure and shear
  - Use when distance from face-of-support to 0-shear is < 2d
  - Use when a girder load is within 2d to face-of-support
  - In other words, use when plane-sections don’t remain plane!
- Common applications: integral bent caps, C-bent caps and footings, outrigger bents
- Requires “thinking outside of the box”!

Where are plane-sections-remaining-plane?

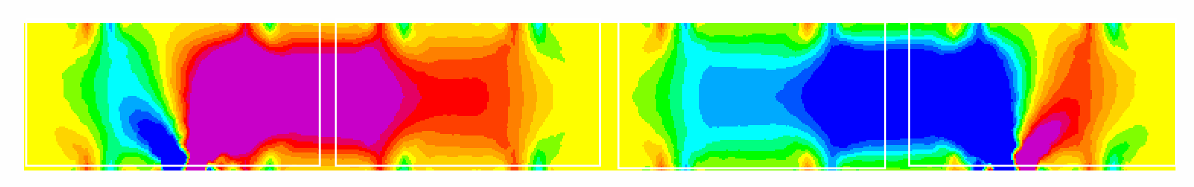
LIVE LOAD

3 D MODEL

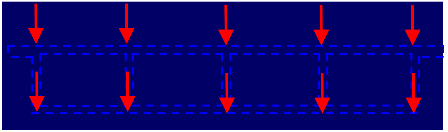




2 D MODEL



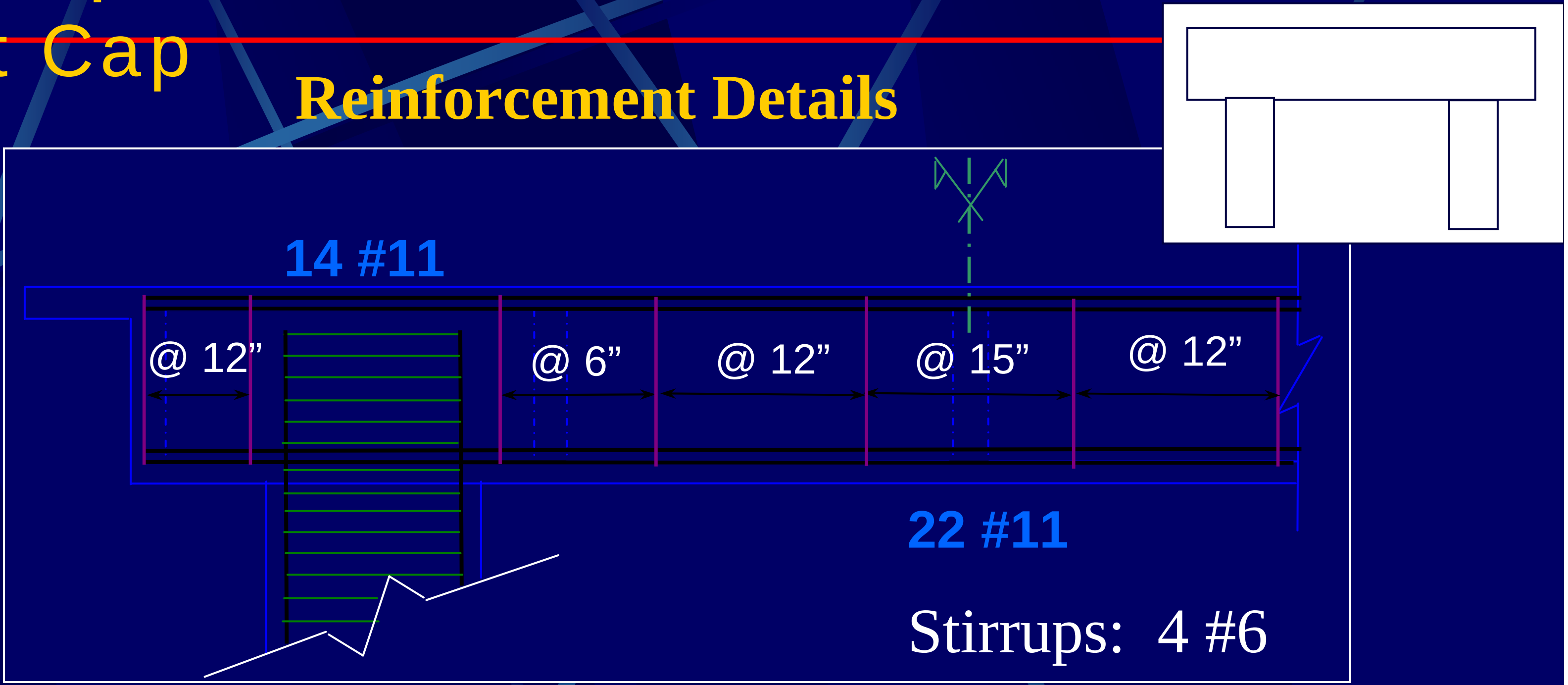
Equally Divided



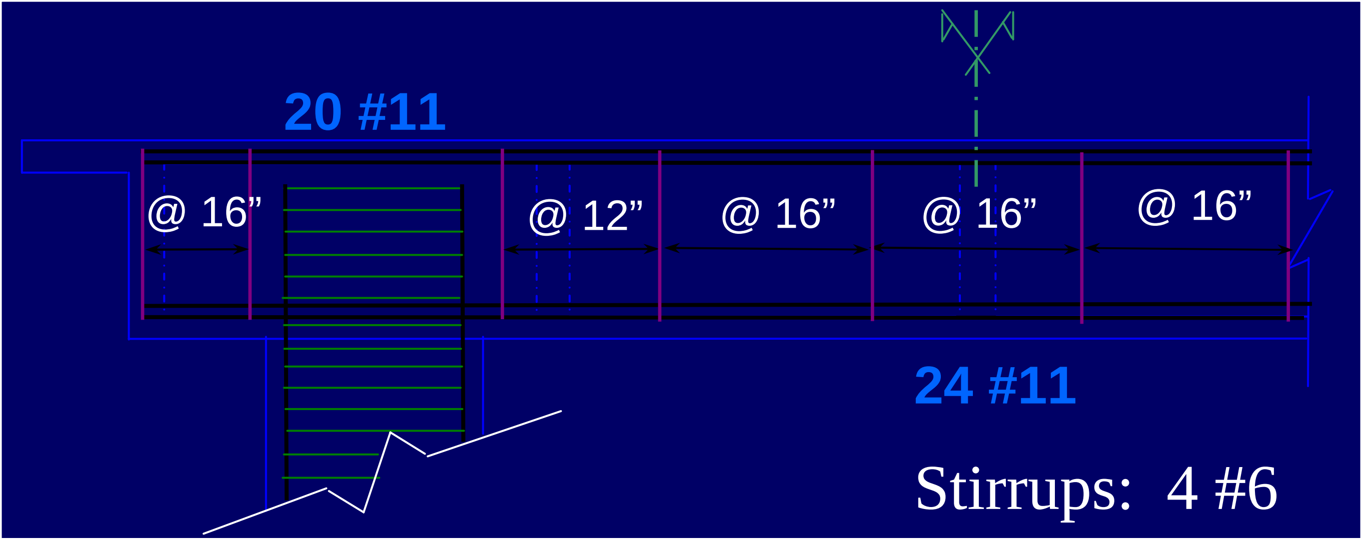
Example: Two – Column Integral Bent Cap

Reinforcement Details

STM



SM



Seismic Shear

$V_c = v_c A_e$ where $A_e = 0.8A_g$

$$v_c = 0.032\alpha'\left(1+\frac{P_u}{2A_g}\right)\sqrt{f'_c} \le \min\left\{\begin{matrix}0.11\sqrt{f'_c}\\ 0.047\alpha'\sqrt{f'_c}\end{matrix}\right.$$

$$0.3 \le \left\lfloor \alpha' = \left(\frac{f_{s\ or\ w}}{0.15}+3.67-\mu_D\right)\right\rfloor \le 3;$$

$$f_s\left(spiral, hoop\right) = \rho_s f_{yh} \le 0.35; \quad \rho_s = \frac{4A_{sp}}{sD'}$$

$$f_w\left(rectagle\right) = 2\rho_w f_{yh} \le 0.35; \quad \rho_w = \frac{A_v}{bs}$$

Torsion—is it right?

- Before 3rd Ed., ’05 Interims
  - Specs were silent on modifying V_u for T_u
  - For β–θ table, v_u and ε modified for T_u
  - Result: lower values for β and hence V_c
- Changes 3rd Ed., ’05 Interims
  - V_u modified for T_u
  - Silent on modifying v_u and ε for T_u

Conclusions and Recommendations

- Consider which methodology will be used for rating or issuing permits; then set design policy
- Strut-and-tie is more appropriate for knee joints and loads from framed gdrs; look for NCHRP Report in the future
- For columns subjected to extreme events, see the new Seismic Guide Specs

Acknowledgements

- Ahmed Ibrahim, Caltrans
- The AASHTO SCOBS T10 Committee